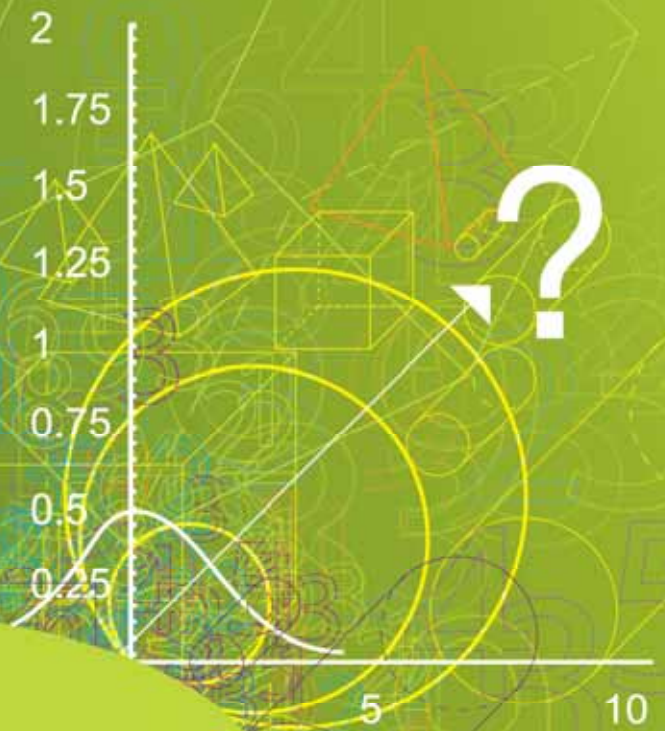
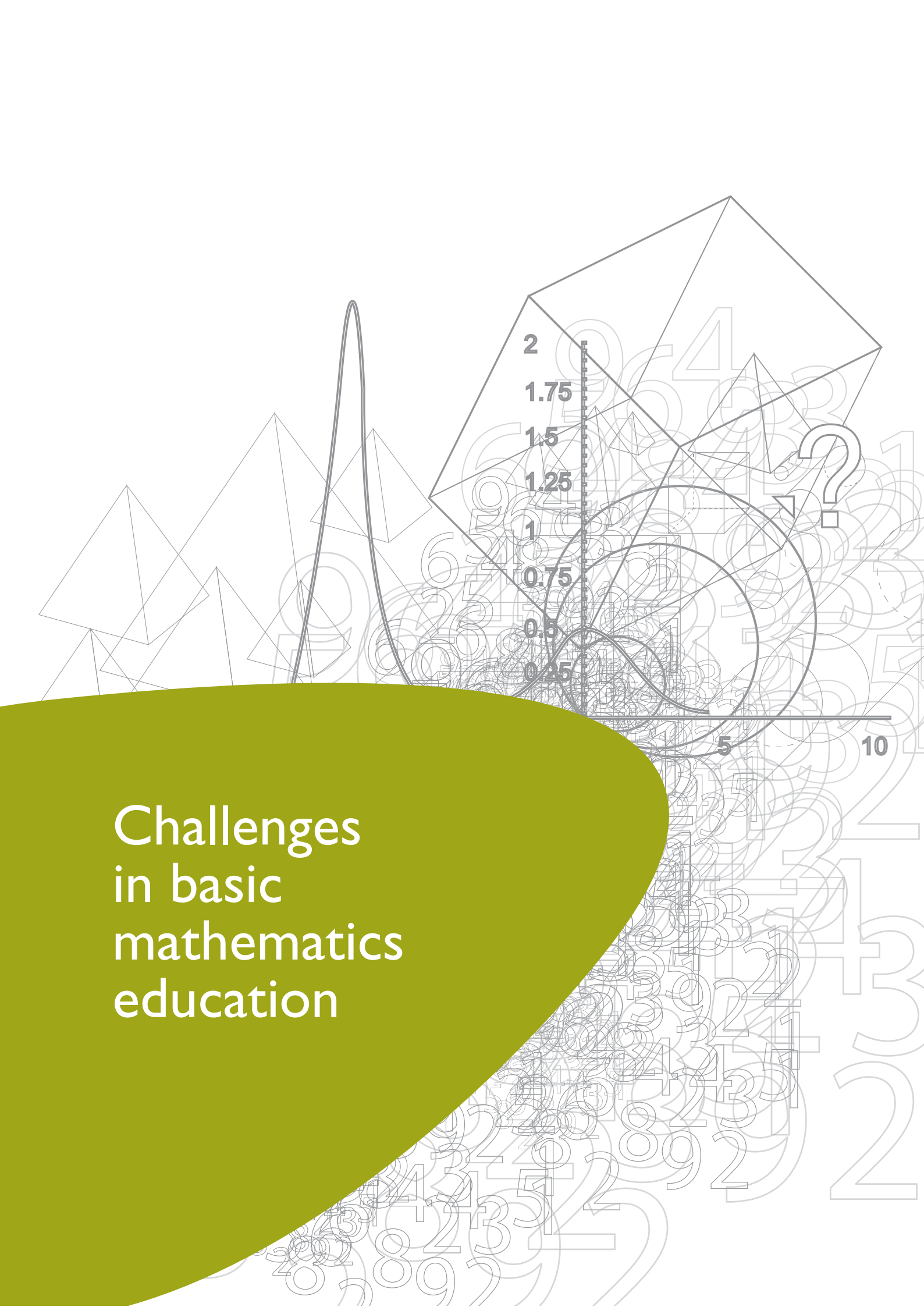




United Nations  
Educational, Scientific and  
Cultural Organization



# Challenges in basic mathematics education

The background features a complex collage of mathematical elements. On the left, there are several overlapping triangles and a tall, thin, curved shape resembling a parabola. In the center, a 3D wireframe pyramid is superimposed over a 2D coordinate system. The y-axis of this system is labeled with values 0.25, 0.5, 0.75, 1, 1.25, 1.5, 1.75, and 2. The x-axis is labeled with 5 and 10. A large question mark is positioned to the right of the coordinate system. The entire scene is overlaid on a dense field of faint, overlapping numbers and mathematical symbols. A large, solid green shape with a curved edge is in the bottom-left corner, containing the title text.

# Challenges in basic mathematics education

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# Foreword

Our world is profoundly shaped by science and technology. Preserving the environment, reducing poverty and improving health: each of these challenges and many more require scientists capable of developing effective and feasible responses – and citizens who can engage in active debate on them.

In order to achieve this, the 1999 Budapest Declaration underlined the importance of science education for all. Indeed, science and mathematics education (SME) that is relevant and of quality can develop critical and creative thinking, help learners to understand and participate in public policy discussions, encourage behavioural changes that can put the world on a more sustainable path and stimulate socio-economic development. SME can therefore make a critical contribution to the achievement of the Millennium Development Goals adopted by the world's leaders in 2000.

Recognizing this, UNESCO created the International Group of Experts on Science and Mathematics Education Policies, whose first meeting on SME in basic education was held from 30 March to 1 April 2009. The conclusions from this meeting, which form the basis for this publication, show remarkable consensus on the challenges faced by SME today and how these can be addressed. All the experts agreed that the last decade has witnessed the development of a substantial body of knowledge on SME and the production of valuable tools and resources, many of which are now widely accessible thanks to technological advances. These are a firm basis to build on and open new perspectives for evidence-based policy for SME.

This publication therefore defines the challenges faced in the implementation of quality SME in basic schooling and, using case studies, sets out ways of improving its delivery. It will be of use not only to decision-makers wanting to mainstream quality SME education into their systems, but also to stakeholders who wish to participate in the change process.

UNESCO hopes that this publication will help mobilize the energy and enthusiasm of children, teachers and parents for improving mathematics education. Indeed, working together on developing quality basic SME in a sustained and coordinated way is the *sine qua non* for ensuring a fairer and more sustainable future for all.

Qian Tang  
Assistant Director-General for Education

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Moreover, UNESCO wishes to express its gratitude to Bill Barton, President of ICMI, and Bernard Hodgson, Secretary-General of ICMI, for their valuable contribution.

# Table of Contents

Foreword	3	<b>7. Organized complementarity between formal and non-formal education</b>	31
Acknowledgements	5	<b>8. Change management and adjustment</b>	33
<b>1. Introduction</b>	9	<b>9. The technological challenge</b>	35
<b>2. Mathematics education and literacy</b>	13	<b>10. Collaboration</b>	37
<b>2.1</b> The challenge of mathematical literacy	13	<b>11. The diversity challenge</b>	39
<b>2.2</b> Beyond the development of mathematical literacy	15	<b>11.1</b> Language issues	39
<b>2.3</b> Content acquisition/Competency development	17	<b>11.2</b> Gender issues	40
<b>2.4</b> Mathematics education for all/ Quality mathematics education	18	<b>12. The challenge of research</b>	41
<b>3. The challenge of changes in teaching practices</b>	21	Summary	43
<b>4. The challenge of assessment</b>	23	References	45
<b>5. Teacher-related challenges: status, initial and in-service training</b>	25	Annexes	51
<b>5.1</b> The quantitative challenge	25	<b>Annex 1.</b> Connections between mathematics and science education in the German SINUS programmes	53
<b>5.2</b> The qualitative challenge	26	<b>Annex 2.</b> Forty years of working on mathematics education, seeing mathematics as a human activity for all – The Freudenthal Institute	56
<b>6. Achieving synergy among all</b>	29	<b>Annex 3.</b> Problems and Challenges in Mathematics Education: The Philippines Context	61
<b>6.1</b> Broader and better acknowledged involvement of mathematicians	29	<b>Annex 4.</b> The professional development of teachers in Japan – the concept of Lesson Study	63
<b>6.2</b> Better inter-community collaboration	30		

<b>Annex 5.</b> The Professional Development of Mathematics Teachers in Brazil: structural issues, initiatives and hopes	66
<b>Annex 6.</b> Systematizing knowledge about Mathematics Teacher Education – The IEA Teacher Education and Development Study in Mathematics (TEDS-M)	71
<b>Annex 7.</b> Research on mathematics teacher education in South Africa and Southern Africa	73
<b>Annex 8.</b> Promoting excellence in mathematics teaching – The National Centre for Excellence in the Teaching of Mathematics (NCETM)	76
<b>Annex 9.</b> Experiencing Mathematics! – An international travelling exhibition	80
<b>Annex 10.</b> Challenging Mathematics – Mathematics Houses in Iran	84
<b>Annex 11.</b> Collaboration between mathematicians, teachers and researchers in mathematics education – the case of the IREM network	87
<b>Annex 12.</b> The emergence of a teacher community – the case of Sesamath	89
<b>Annex 13.</b> Fostering interactions and collaborations “Teacher Education Around the World: Bridging Policy and Practice”, a component of the IAS/Park City Mathematics Institute, Institute for Advanced Study, Einstein Drive, Princeton, New Jersey	92
<b>Annex 14.</b> Rebuilding a mathematical community in Cambodia	95
<b>Annex 15.</b> List of participants at the Experts’ Meeting	98

# I. Introduction

Science education and mathematics education have many values in common and largely face the same problems and challenges. However, there are differences between these two fields of learning, and so there are two separate texts. In particular, it is indisputable that, unlike science, mathematics must be taught to all pupils from the beginning of compulsory education. Mathematics education is not necessarily provided in a satisfactory manner, but it is accessible to all pupils enrolled in school.

It is generally agreed that mathematics must be taught during basic education, but this does not mean that mathematics education itself is not a subject of debate. Both national and international evaluations show that, on completion of basic education, many pupils' mathematics knowledge and competencies fall short of the expected level.<sup>1</sup> Moreover, the disparities observed between and within countries give cause for concern. Even among pupils who obtain satisfactory evaluation results, many do not like mathematics and do not see the point of spending so much school time on the subject.<sup>2</sup> These findings indicate that the goals stated in the introduction are far from being achieved and that the large number of young people who lack access to education is not the only obstacle to their attainment, even though it is a real obstacle.

In this context, the expected outcome of quality mathematics education for all is not self-evident and is the subject of continuing debate. It is thus important to state our position on this point, taking into account in approaches to mathematics and teaching methods those matters that often make quality mathematics education for all problematic.

It is generally agreed that mathematics must be taught during basic education, but this does not mean that mathematics education itself is not a subject of debate.

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- 1 From an international perspective, see the results of Trends in International Mathematics and Science Study (TIMSS) organized by the International Study Centre (ISC), the Programme for International Student Assessment (PISA) organized by the Organisation for Economic Co-operation and Development (OECD) and the Second Regional Comparative and Explanatory Study (SERCE), conducted by the Latin American Laboratory for the Assessment of Quality in Education (LLECE) in Latin America.
  - 2 This phenomenon has been identified by TIMSS and PISA international evaluations in particular for pupils from Asian countries.



Everyone recognizes, for example, that mathematics is omnipresent in today's world – notably in the technological items all around us and in exchange and communication processes – but it is generally not in evidence.

Everyone recognizes, for example, that mathematics is omnipresent in today's world – notably in the technological items all around us and in exchange and communication processes – but it is generally not in evidence, which makes it difficult for some to see the point of developing a mathematics culture beyond basic numeracy, measurements and calculation. It is important for basic education to help to bring mathematics to the fore, especially because “mathematical literacy” requirements far exceed needs traditionally associated with basic computational knowledge. This point will be considered later in the document (see Section 2).

Many misunderstandings also affect people's view of mathematical activity, owing to their perceived image of mathematicians. Mathematics is still often perceived as an almost exclusively solitary activity, cut off from the problems of the real world and independent of technology. Furthermore, it is often still seen as a purely deductive activity in which perfectly rigorous formal proofs are used to produce theorem after theorem. Finally, it is often considered that mathematics is a science that is not for everyone and that girls, in particular, are likely to encounter more difficulties than boys

in learning mathematics.<sup>3</sup> These many misunderstandings affect teaching and raise barriers to quality mathematics education for all.

Quality mathematics education should enable pupils to form a positive and appropriate image of mathematics. For that to be possible, it must be faithful to mathematics, both in its content and practices. It should enable pupils to understand which needs are met by the mathematics that they are taught and that mathematics forms part of a long history linked to the history of humanity.<sup>4</sup> Learning mathematics also entails acquiring the means of gaining access to this cultural heritage. Mathematics education should thus enable pupils to understand that mathematics is not a static corpus of knowledge but, on the contrary, a living and expanding science, whose development is nourished by that of other scientific fields and nourishes them in turn. It should also enable pupils to see mathematics as a science that can and must contribute to the solution of today's major world problems, which were mentioned in the joint introduction. Quality mathematics education must thus be sustained by a vision of mathematics as a living science, grappling with the real world, open to relations with other disciplines and not confined to scientific disciplines

3 On gender issues, see the long list of references accessible on the website of the International Organisation of Women and Mathematics Education (IOWME) at <http://extra.shu.ac.uk/iowme/>. These issues will be addressed in Section 11.2.

4 In regard to the historical aspect, see the work done by the History and Pedagogy of Mathematics (HPM) international group at <http://www.clab.edc.uoc.gr/HPM/> and the study commissioned on the subject by the International Commission on Mathematical Instruction (ICMI) (Fauvel and van Maanen, 2000).

only. In particular, it must enable pupils to understand the power of mathematics as a tool for moulding understanding and influencing the world.<sup>5</sup>

Quality mathematics education must, moreover, faithfully portray the practices of those who produce or use mathematics. Mathematical activity is a multifaceted human activity, very different from the stereotypes often attached to it in popular culture. Quality mathematics education must therefore reflect that diversity in the different mathematics content gradually encountered by pupils, by setting or reformulating problems to make them amenable to mathematics, by modelling, exploring, conjecturing, testing, representing and formulating – using specific vocabulary – arguing and proving, developing methods, working out and connecting concepts within structured spaces, exchanging and communicating. Such an education should make it possible for mathematics to be both an individual and a collective life experience and for the value of communication and exchange with others to be perceived. It must stimulate by setting challenges while cultivating values of solidarity. It must also reflect education open to the world and thus in tune with non-formal scientific and social mathematics practices and capable of using in a relevant manner the technological tools employed in those practices.

Bringing mathematics education into line with these values, in an education for all contexts, is a challenge that education systems must take up if mathematics education, coherent with and complementary to sciences education, is to contribute as it must to scientific, economic and social development, citizenship and personal fulfilment. In taking up this challenge, substantial changes must be made to the current state of mathematics education. Emphasis will be laid in the remainder of this document on the changes that seem to be most crucial and some conditions required to achieve such changes will be noted. An attempt will also be made to show that such developments are possible, drawing on achievements in various economic, social and cultural contexts. On the basis of these examples, it can be stressed that, while common principles can guide action, there is no single path to progress and no solution that can be transposed directly from one educational context to another. Finally the examples show that, to achieve positive and sustainable improvements, there must be continuity of political action over time, founded on organized collaboration among all actors involved and forms of action that break with usual practice, thus ensuring that initiatives and responsibilities are appropriately assigned.

Mathematical activity is a multifaceted human activity, very different from the stereotypes often attached to it in popular culture. Quality mathematics education must therefore reflect that diversity in the different mathematics content gradually encountered by pupils.

<sup>5</sup> As to modelling, see the work done by the International Community of Teachers of Mathematical Modelling and Applications (ICTMA) international group at <http://www.ictma.net> and the ICMI Study on the subject (Blum *et al.*, 2007).

## 2. Mathematics education and literacy



Mathematical literacy for all young people is not only the goal, but also the fundamental priority objective of mathematics taught during basic education.

Once imparted, numeracy enables the development of the mathematical knowledge and competencies necessary for integrated and active participation in a given society and for adaptation to foreseeable changes in that context. It gives access to a world broader than that in which young people were educated and prepares them to find their place in today's world, to develop in it and to help to take up the great challenges facing humanity today in the fields of health, environment, energy and development. This goal is far from being achieved and constitutes a primary challenge for basic mathematics education.

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### 2.1 The challenge of mathematical literacy

The challenge to be taken up first of all is that of access to basic education. Millennium Development Goal No. 3, namely access to basic education for all young people by 2015, is far from being achieved. Today 75 million children are still not enrolled in primary school. Universal access raises, in particular, the quantitative challenge of the availability of a sufficient number of qualified teachers (see Section 5). It cannot be minimized. In this paragraph, however, we would like to focus on another challenge, that of adapting basic education to meet current expectations in terms of mathematical literacy. As mentioned above, these expectations have risen considerably because of technological, economic and social changes, and they will continue to rise in the future.

Mastery of basic numeracy and measurement, which has long constituted the rudiments of mathematical knowledge required for participation in society, no longer suffices today. Owing to the digital culture that is increasingly embedded in our contemporary societies, the new responsibilities that individuals must shoulder personally or as citizens and the growing

Mastery of basic numeracy and measurement, which has long constituted the rudiments of mathematical knowledge required for participation in society, no longer suffices today. Owing to the digital culture that is increasingly embedded in our contemporary societies, the new responsibilities that individuals must shoulder personally or as citizens and the growing uncertainty of the world in which we live, the idea of mathematical literacy must be revised.

uncertainty of the world in which we live, the idea of mathematical literacy must be revised. Traditionally, the content of mathematics education for all has consisted of numeracy, the decimal system, arithmetic operations and the capacity to solve elementary arithmetic problems such as problems of proportionality, knowledge of systems of magnitudes and knowledge of common two- and three-dimensional geometrical forms. These remain the core bases of mathematical

literacy. As in the past, children must learn the significance of numbers and formulae, learn to estimate, to measure and to play with orders of magnitude. However, on the one hand, these bases are no longer sufficient to meet today's growing needs and, on the other hand, they cannot be taught without taking into account the social conditions in which this knowledge is used today and the new learning resources heralded in by technology.

Today, mathematical literacy must, in particular, make it possible for individuals to understand, analyse and critically assess multiple data delivered by various complex systems of digital, symbolic and graphical representation – most often interactively. It must enable them to make reasonable choices based on comprehension, modelling and prediction and to ascertain their effects in new situations often fraught with uncertainty. It is thus essential that all pupils learning mathematics during basic education are gradually exposed to the complexity of the current digital world, learn to position themselves in order to act in that world and become familiar with the diversity of the modes of representation that it uses. It is important for the pupil to become gradually familiar with probabilistic and statistical modes of reasoning required for mathematical thought in order to understand phenomena that, in both science and social life, involve uncertainty and risk.

As stressed above, account must also be taken of the actual and potential educational use of current technologies. Their actual use has given rise, above all, to an undeniable change in the calculation practices in society. Calculation has always been a key component of mathematical literacy, but it is increasingly performed by a variety of tools. Therefore, to organize and check their calculations, people must have greater capacity for estimation, reasoning based on the properties of numbers and operations, and new balances between exact and approximate calculations and between written and mental calculations. If pupils in basic education are to be trained appropriately in current forms of calculation, our vision of learning and, in particular, the goals set for learning arithmetic techniques must be reconsidered. To this question, a source of inexhaustible debate, there is obviously no uniform

answer independent of the contexts and resources that are socially available for mathematical activity.

Furthermore, mathematics education is not the sole contributor to the development of the knowledge necessary for mathematical literacy but, in so contributing, it must interact closely with the teaching of other disciplines, particularly scientific disciplines, in order to close the gap between them, as highlighted in the joint introduction to the two texts on mathematics education and science education. Mathematics education nevertheless plays a key role in this area because it is the only discipline in which the subjects and techniques concerned are studied as such and in which the progression of knowledge thereon is organized systematically. This point of view is outlined, for example, in *Mathematics and Democracy. The Case for Quantitative Literacy* published in 2001 by the National Council on Education and the Disciplines in the United States of America (Steen, 2001), which also acknowledges that this mission is not fulfilled in mathematics education in that country. Furthermore, it stresses that the concept of mathematical literacy should not be regarded as fixed, independent of time and space. The stated quantitative literacy requirements are very obviously those of today's American society or of societies comparable in terms of development and societal choices. However, without minimizing cultural differences, it seems important to stress that the changing and growing need for mathematical literacy observed worldwide must be taken into account when designing quality mathematics education for all. Moreover, it must be remembered that the mathematical literacy component of basic mathematics education must make it possible to anticipate future societal changes and to open the door to other worlds.

## **2.2 Beyond the development of mathematical literacy**

Even in basic education, the sole ambition of quality mathematics education for all cannot be reduced to the development of mathematical literacy as defined above. Mathematics education, even during compulsory education, must also meet other needs. It must enable everyone to perceive the incredible human adventure inherent in the centuries-long development of mathematics in all continents, an adventure inseparable from the history of humanity. It must empower everyone to ask questions about the role that mathematics has played and still plays in scientific, technological, economic and social development. It must enable pupils to practise modes of mathematical thought such as abstraction, generalization, logical reasoning, proof and mathematical symbolization and to understand the power of such thinking. It must also prepare for the further training of those who will enter professions requiring knowledge of advanced mathematics and it must kindle young people's interest in these professions. This poses a real challenge in many countries.

It is therefore important to portray mathematics as a living science, rooted in the world and interacting with other scientific fields. Account must therefore be taken of some characteristics of contemporary mathematics mentioned by László Lovász, president of the International Mathematical Union, at the conference held in Lisbon in 2007 on the future of mathematics education in Europe. These characteristics are the exponential growth of the mathematics community and of research activities in this field, the new areas of application of mathematics and their growing influence, the new mathematical tools provided by computers and information and communication technology, and new forms of mathematical activity (Lovász, 2007). Consideration must be given, in particular, to the interfaces of mathematics other than its historical interaction with physics, such as its interfaces with computer science, economics and biology, internal developments evinced within mathematics itself by the growing importance of fields such as discrete mathematics and probability, and trends in interaction among mathematical fields. Account must be taken of changes in mathematical practices closely related to technological change, as evidenced by the importance and rising profile of experimental mathematics, the technological support for calculation, visualization and simulation, the strengthening of and new approach to the algorithmic dimension of mathematics, the reasoned and effective management of the current diversity of information sources and possible modes of collaborative work.

How can account be taken of these changes in basic education? In view of the diversity of current mathematics, choices must be made. As László Lovász stressed at the above-mentioned conference on mathematics education generally, the choices to be made are not easy and are further compounded by a context in which the general trend is to reduce the number of hours set for mathematics education. The choices are even harder when they concern basic education in which pupils learning mathematics have only limited and often fragile knowledge. However, to avoid perpetuating the already widespread idea among pupils that mathematics is a dead science, the challenge of striking a satisfactory balance between the development of the mathematical competencies expected of all and openness to selected current issues must be taken up. These changes must be made without opposing traditional and contemporary mathematical ideas, reconsidering the teaching of traditional subjects to ensure that they reflect contemporary mathematical views and practices more effectively, and improving interaction between mathematics education and science education. There is no single solution to this problem, but it is important to make coherent and realistic choices, taking the diversity of contexts and cultures into account.<sup>6</sup> These choices must be informed by a vision of recent developments in mathematical sciences, viewed in terms of their likely educational implications. That vision must be transmitted to teachers in appropriate ways, and the Felix Klein Project,

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6 For an example of such reflection in the French context, see the book (Kahane, 2001) produced as a result of the work of the Commission for Reflection on Mathematics Education (CREM) in France and the various documents written by the commission and available on the website of the French Mathematical Society at [smf.emath.fr/Enseignement/CommissionKahane/](http://smf.emath.fr/Enseignement/CommissionKahane/).

launched jointly by the International Mathematical Union and the International Commission on Mathematical Instruction, was designed to achieve that goal.<sup>7</sup>

### 2.3 Content acquisition/Competency development

In view of the above considerations, questions must be raised about both educational content and specific learning expectations in respect of such content. Operational knowledge, defined as the capacity to draw on mathematical tools to deal with new and potentially problematic situations, rather than merely the capacity to reproduce procedures learnt in relatively stable contexts very similar to those in which they were taught, is the primary agreed expectation today. It has also been generally agreed that knowledge must be sufficiently solid and structured if it is to be a basis for later learning, in view of the cumulative character of mathematical knowledge. Reflection in this area has been backed by systematic efforts to express what is meant by mathematical competence, in an attempt to determine categories that transcend any particular content to contribute to a more broadly based understanding of mathematical thought and its possible progression. For example, Kilpatrick, Swafford and Findell (2001) define what they call “mathematical proficiency” as the result of the entwining of five features, namely “conceptual understanding, procedural fluency, strategic competence, adaptive reasoning and productive disposition”. In the KOM model devised in Denmark (Niss, 2002), which informed that country’s reform of secondary education in 2005 and inspired the concept of “mathematical literacy” in the OECD PISA programme (OECD, 1999, 2006), mathematical competence is defined as insightful ability to act appropriately in somewhat mathematically challenging situations. Eight major distinct, but not independent, competencies have been identified.<sup>8</sup> The extent to which each one is developed in a given individual is evaluated

Operational knowledge, defined as the capacity to draw on mathematical tools to deal with new and potentially problematic situations, rather than merely the capacity to reproduce procedures learnt in relatively stable contexts very similar to those in which they were taught, is the primary agreed expectation today.

7 The International Commission on Mathematical Instruction (ICMI) is a commission of the International Mathematical Union. For more information on the Felix Klein project, see the ICMI website at <http://www.mathunion.org/ICMI/>. This project currently concerns mainly senior high school teachers but is to be extended to all mathematics teachers.

8 The transverse competencies are the following: (1) thinking mathematically (mastering mathematical modes of thought); (2) posing and solving mathematical problems; (3) modelling mathematically (i.e. analysing and building mathematical models); (4) reasoning mathematically; (5) representing mathematical entities; (6) handling mathematical symbols and formalisms; (7) communicating in, with and about mathematics; and (8) making use of aids and tools (IT included).

under three heads, namely mastery of its characteristic features, the extent of applicable contexts and situations and the technical level of these applications.

Owing to the attention paid to explaining cross-cutting competencies, in various countries there has been a shift away from curriculum descriptions with the traditional emphasis on content to curriculum descriptions structured around the acquisition of such competencies. It is now considered important to find a reasonable balance and connection between these two types of description in defining basic mathematics education. The usual definitions in terms of content alone are generally implicit about the exact competence expected at the end of educational stages and do not explain clearly how specific learning in the various areas contributes to the more general objective of developing mathematical competencies. They do not, therefore, facilitate the necessary changes and adaptations mentioned above. However, definitions of general competencies do not suffice on their own to build a coherent curriculum that respects the epistemology of the fields concerned, highlighting the rationale behind the concepts and techniques taught and taking account of the cumulative character of mathematical knowledge. As underlined by Winslow (2005), mathematics is the outcome of human history in which the idea of developing cross-cutting competencies makes little sense. Curriculum development for basic education must thus strike a balance in combining two complementary approaches – one in terms of content and the other in terms of cross-cutting competencies. This is a real challenge because experience has shown that it is difficult to strike a satisfactory balance. In particular, it is important to show clearly the way in which the teaching of specific mathematical topics contributes to the development of cross-cutting competencies without blurring the specificity of these contributions. For example, in mathematics, forms of reasoning and proof are, beyond common logical bases, narrowly dependent on the areas in which they develop. The effectiveness of reasoning does not rest on the same schemas in number theory, geometry or probability and statistics.

## **2.4 Mathematics education for all/Quality mathematics education**

Basic education must, as stressed earlier, provide quality mathematics education for all pupils. These two goals – provision of quality mathematics education and provision of mathematics education for all pupils – are often perceived as irreconcilable. They are objectively irreconcilable if there is a shortage of qualified teachers to ensure, under satisfactory conditions, universal access to basic education, as in many developing countries, unfortunately. Moreover, behind this view, there is often the idea that quality mathematics education is necessarily selective and that provision for all pupils perforce entails lower quality. Mathematics education must take up the genuine challenge of dispelling this view, often firmly rooted culturally – and this view is far from



being dispelled. However international evaluation results (OECD, 2004, 2007) tend to show that the most successful education systems include those which have opted for inclusive basic education. The diversity of the educational choices open to the countries concerned shows, once again, that there is no single solution to this problem. It also shows that a growing number of comparative studies, often motivated by international evaluations, can improve our understanding of the options offered (see Kaiser, Luna and Huntley, 1999; Leung, Graf and Lopez-Real, 2006). Lastly, it must be stressed that inclusive basic education does not exclude organizing extra-curricular activities, as in many countries, to strengthen pupils' interest in mathematics and to accommodate those who wish to be more intensely involved (see Section 7).

Basic education must, as stressed earlier, provide quality mathematics education for all pupils. These two goals – provision of quality mathematics education and provision of mathematics education for all pupils – are often perceived as irreconcilable.

### 3. The challenge of changes in teaching practices

**T**o overcome the above-mentioned challenges, changes in teaching practices must be made consistently with the stated goals. Research on classroom practices in the context of teaching and training studies and surveys conducted by international institutions (European Commission, 2007) show that, for the moment, this is not generally the case. Basic mathematics education is still all too often boring because:

- 
- it is designed as formal teaching, centred on learning techniques and memorizing rules whose rationale is not evident to the pupils;
  - pupils do not know which needs are met by the mathematics topics introduced or how they are linked to known concepts;
  - links to the real world are weak, generally too artificial to be convincing and applications are stereotypical;
  - there are few experimental practices and modelling activities;
  - technology is quite rarely used in a relevant manner;
  - pupils have little autonomy in their mathematical work and often merely reproduce activities.

It has been shown in a growing number of investigations and experimentation over the years that alternatives, productive in terms of learning, do exist and give pupils another view of mathematics and of their capacity to grasp the significance of this science (see, for example, Bishop et al., 1996, 2003; and Lester, 2007 for synoptic views). Such alternatives are generally based on socio-constructivist approaches to learning (Ernest, 1999) and they stress the role of problem-solving in the teaching of mathematics, whether those problems are used to motivate pupils and prepare them for the introduction of new concepts or to enable them to study and apply concepts already introduced. Learning is perceived as a gradual process, through which meaning is built up by comparing meeting carefully selected problematic situations and by drawing on diverse systems of representation and devices, since mathematical objects are not directly accessible to the senses. The social dimension of such learning, through interaction among pupils and between teachers and pupils, is heavily emphasized, as is the importance of the pupils' out-of-school experience.

Many studies also show, however, that when teachers attempt to adapt their practices to the predominant socio-constructivist discourse, for example by setting more open problems for the pupils, ostensibly to engage them in investigative practices, the results are not necessarily satisfactory. It has often been observed that the pupils' activity, even when suitably targeted and reasonably productive mathematically – which is not necessarily the case – is only exploited with difficulty by the teacher if he or she has not been specifically trained. The sharing of mathematical responsibilities between teachers and pupils implied by this view of learning is in fact far from easy. It requires suitable tasks and guidance for the pupils and a suitable didactic contract (Brousseau, 1997). It requires teachers capable of dealing with the unexpected and of identifying the mathematical potential of pupils' ideas and work that have not necessarily been anticipated. Finally, it requires teachers capable of helping pupils to link their results in a particular context to knowledge targeted by the institution in terms of both content and form. The teaching expertise required is thus much greater than that required in traditional teaching practices.

This raises the inevitable question of teacher training and of the resources at their disposal to enable them to improve their practices. This subject will be raised later in this text, but some points must be stressed now: in particular, suitable professional training should assist teachers more effectively in designing tasks likely to permit mathematically productive investigations within the constraints of the classroom and should help teachers to play their role as guide and mediator in managing these tasks in a mathematically effective manner. Furthermore, changes to practices must be considered dynamically, with care being taken to establish a reasonable distance between old and new practices, and must be constantly supported by appropriate resources to initiate and support the desired changes. Teachers, in pre-service training or once resources are placed at their disposal, are all too often presented with models of practices that are too different from their actual experience to be assimilated without being distorted. Furthermore, the increase in the mathematical and pedagogical expertise required by new practices is greatly underestimated. This makes it difficult for teachers to perceive the benefits of recommended changes and they are therefore not motivated to put them into practice.

These considerations call into question the appropriateness of models used for teacher training and for the dissemination of innovations and research – which will be addressed later.

It will first be stressed, however, that although the socio-constructivist model briefly described above more or less explicitly inspires many of today's innovations and educational activities, it can take very different forms, depending on the social and cultural context. Moreover, it is not the only possible model (Lerman and Sierpinska, 1996). This has been shown in studies such as the ICMI study which compares the education culture in Asian countries of a Confucian tradition with that of Western countries (Leung, Graf and Lopez-Real, 2006), and The Learner's Perspective Study (Clarke, Keitel and Shimizu, 2006), (Clarke, Emanuelsson, Jablonka and Chee Mok, 2006), which compares the practices of teachers recognized as experts in twelve countries.

## 4. The challenge of assessment

**A**ssessment is of the essence in mathematics education, both formatively – to guide learning incrementally, and summatively – to compare the results obtained with those expected and then to evaluate the difference between actual achievements and the curriculum set. For that purpose, assessment must combine internal, external, quantitative and qualitative factors and rely on suitable methodologies and tools. This is the general consensus, on which there is no need to dwell.

A crucial question in this field is that of consistency between the means of assessment and educational goals, in respect of their underlying values. Such consistency is crucial, in view of the influence that assessment exerts on teaching, and poses a real challenge for mathematics education. It is not easy to achieve for; as underlined above, quality mathematics education has various aims, in terms of knowledge, specific and cross-cutting competencies and attitudes towards the discipline. It brings into play individual capacities but also capacities of a more collective nature. It must take into account the fact that sufficient time must be allowed for problem-solving, which represents an essential feature of mathematical activity, if it is to be evaluated properly. It must be consistent with the practices relating to authorized technological tools. Furthermore, from the UNESCO point of view of quality basic education accessible to all as a means to fulfilment and self-development, it must be designed to enable each pupil to express his or her knowledge and competencies as well as possible, while being attentive to the diversity of the potential forms of such knowledge and competencies.

All this argues in favour of a multiform assessment, as no single form of assessment can meet all of these conditions. In particular, it is important to recognize that research activities, experimental activities, the conduct of mathematics projects, syntheses, oral presentations, work of a historical nature and practical achievements all have their place in quality mathematics education for all. They must thus be evaluated so that their importance can be recognized institutionally, but the forms of assessment must be tailored to each case.

There is currently a strong trend towards conducting many assessments and, with the emphasis on quality scientific assessment, to conduct them on a large scale and to minimize their costs

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by basing them on a series of questions with multiple choices or short answers, processed automatically if possible.<sup>9</sup> (The French meaning of this sentence is changed by the translation. The idea expressed in the French text is that the desire of ensuring scientific quality of assessment and large scale implementation at a minimal cost leads to base assessment ....) Such assessments may be very well designed and may be the source of interesting information, as many examples show. However, they limit the appraisal of potentially quality mathematics education by reducing the assessment to what the tools, constrained as they are, can actually assess. It therefore seems dangerous to use only these modes of assessment or to make them the principal instruments of managing an education system. Recent history contains examples of the perverse effects of such devices (Schoenfeld, 2007), (Keitel, 2008), showing, in particular, that, in the most fragile contexts, teaching can become “teaching to the test”, which, whatever the quality of the test, is irreconcilable with quality mathematics education as generally defined.

Assessment has a crucial role to play in establishing and successfully providing quality mathematics education for all. It must serve this cause. Its adaptation to mathematics education values, its quality and direct and indirect effects should be monitored carefully.

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9 Note that large-scale assessments do not all take this form.

## 5. Teacher-related challenges: status, initial and in-service training

**T**eachers are the key to the positive and sustainable development of education systems. They now constitute the principal challenge to quality mathematics education for all. The problems are manifold, quantitative and qualitative.

### 5.1 The quantitative challenge

The quantitative challenge does not affect all parts of the world identically. In some countries, the profession of basic education teacher has a good social image – wages are acceptable, if not attractive, and working conditions are good. All of this helps to make the profession attractive. This situation is far from being the norm, even in developed countries, as shown by the serious teacher recruitment and retention problems experienced in a number of such countries (OECD, 2005). The declining interest in mathematics study at university makes the problem more acute, generating a vicious circle (Holton, 2009). However, the main issues arise, concomitantly in developing countries in the form of an unattractive profession, a shortage of secondary-school pupils likely to study mathematics at university and a shortage of trainers to ensure that they are trained. In many of these countries, pupils or trained teachers leave in droves for countries where professional prospects are better. This is particularly the case in a number of African countries, as shown by the study on the state of teacher education in twelve countries, conducted at the behest of the AFRICMEI conference (Adler et al, 2007). The authors of the study add AIDS mortality to the difficulties mentioned above and stress that although the problems encountered affect the teaching profession in general, mathematics teachers are particularly affected because they have many other employment prospects in the country itself or abroad.

Teachers are the key to the positive and sustainable development of education systems. They now constitute the principal challenge to quality mathematics education for all. The problems are manifold, quantitative and qualitative.

The quantitative teacher recruitment and retention problem is thus a major issue and, if it is to be solved, the problems of mathematics education after basic education must be examined. As highlighted in *Mathematics in Africa: Challenges and Opportunities*, a report produced recently by the Developing Countries Strategic Group of the International Mathematical Union for the John Templeton Foundation (DCSG, 2009), “to concentrate on primary education alone will be futile if there are no qualified teachers; there can be no qualified teachers without skilled mentors to teach them”.<sup>10</sup> This finding implies that there will be sufficient pupil flows into senior secondary education and into higher education. To overcoming this challenge, the real importance of the profession must be recognized socially and teachers’ working conditions must be improved. Furthermore systematic efforts must be made to give all teachers access to networks, resources, in-service training, exchanges and collaboration with others.

## 5.2 The qualitative challenge

The second challenge is quality, for it is clear that, in many countries, the quality of teacher education is far from satisfactory, even when there is no quantitative problem. As stressed earlier, expectations of basic education have risen substantially. To meet these growing demands, teachers must be well trained mathematically, didactically and pedagogically. Most basic education teachers, in particular in the beginning classes but in all classes in a number of countries, have experienced difficulties in their own mathematics education and have a negative image of the discipline. They are often general-purpose teachers and their credit hours of science education and, above all, mathematics education account for only a fraction of their training. This context makes their initial and in-service training all the more problematic.

In view of these basic education characteristics, careful thought must be given to the knowledge that must be acquired in order to exercise this profession and to the ways and means by which it can be developed. There is no denying that the exercise of the profession requires in-depth knowledge of the mathematics targeted by teaching. The first important point is that the mathematics taught during compulsory education is no longer confined, as stressed earlier, to the mathematics taught at that level a few decades ago. All too often, no account of these developments is taken in training future teachers in mathematics, and so they are not trained to portray mathematics as a living science interacting with many other educational fields. This is especially damaging if the target of mathematics education, as stressed repeatedly in this document, is to build productive interaction with science education. The second, even more important, point concerns the specific features of mathematics education. Owing to studies

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<sup>10</sup> The importance to be attached to education after the basic education level was also addressed at the conference on “Higher education and research in developing countries”, organized jointly by the Niels Henrik Abel Memorial Fund and the Oslo Centre for Peace and Human Rights, in Oslo, in February 2008: <http://www.dnva.no/c26889/artikkel/vis.html?tid=27509>.

on teacher training and teaching practices, the question of whether sufficient attention is paid in mathematics teacher education to the profession's specific mathematics requirements has been raised (Even and Ball, 2009). It is now generally agreed, that the knowledge required of teachers is not confined to academic mathematical knowledge and, pedagogical knowledge that could be acquired consecutively or concomitantly. Several categories have been proposed to describe the various types of knowledge concerned, all derived to varying extents from the initial model posited by Shulman (1986), distinguishing "content knowledge", "pedagogical content knowledge" and "pedagogical knowledge". An example is the categorization introduced by Ball et al (2005), drawing on many case studies. It distinguishes four categories of knowledge, namely "common content knowledge" (essentially the mathematics knowledge targeted by the curriculum), "specialized content knowledge" (used by the teacher and transcending the curriculum itself), "knowledge of pupils and content" (knowledge concerning the pupils) and "knowledge of teaching and content" (knowledge of teaching and its organization). These authors stress that school mathematics must be viewed as a specific form of applied mathematics that is not learnt automatically from mathematics taught, however thoroughly, at university. To prove this point, they have proposed educational tasks involving decimal numbers and fractions to university mathematicians and expert primary teachers. Teachers' mathematics education must take this specific feature into account.

The second generally agreed point is that linkage of these various types of knowledge must be addressed explicitly in quality teacher training by suitable means and must be updated in practice. Admittedly, all linkages cannot be understood in pre-service training, as shown in research work by Ma (1999) or Stevenson and Steigler (2000), but the process must begin during initial training and it implies the organized collaboration of various types of expertise – mathematical, didactical and pedagogical. In this interlinking of knowledge, which helps to build up the teacher's professional expertise, didactical knowledge has a particular role to play, owing to its position at the interface between the discipline and professional practice.

It is also clear today that teaching is a profession for which initial training, whatever its quality, must be supplemented by regular in-service training, owing to a variety of factors. On the one hand, as underlined above, some relations between the various forms of knowledge and between knowledge and practice cannot be fully meaningful during initial training, as trainees lack teaching experience. Moreover, the teaching of mathematics must be adapted continuously to developments in mathematical sciences and their relations with the outside world, changes in social demands, developments in teaching conditions and resources, especially technological resources, developments in knowledge produced in various research fields associated with teaching and learning. In too many countries today, in-service teacher training is at best "cobbled together" and lacks long-term vision, coherence and continuity with initial training, which compromises very seriously the likelihood of any sustainable improvement in the quality of education. Nevertheless, owing to developments in knowledge of teachers' practices and scope for development (Krainer



and Wood, 2008), (Vandebrouck, 2008), the ways in which teacher education can further the much-needed development of teaching practices, as already stressed in Section 3, and enable teachers to organize quality mathematics activities in the classroom are now better understood.

Broad consensual views and the points on which it is particularly important to target efforts have been highlighted in this part of the text. As noted in regard to practices, there is no single means of achieving progress, and great attention must be paid to contextual and cultural characteristics. Some countries have a tradition of teacher training that integrates various types of required competencies throughout the training course, while in the tradition of other countries, priority is given initially to subject-specific training; pupils are taught by general-purpose teachers throughout compulsory education in some countries, while this is so only during the first few years of education in other countries and, in yet others, pupils are taught by several teachers from the outset. The development dynamics of the various systems cannot be addressed in the same way but, here too, this situation permits quite interesting comparisons that give insights into the strengths and weaknesses of the various systems and render quite conceivable developments that would not have been envisaged within a given system. From this point of view, the recently published ICMI study on *The Professional Education and Development of Teachers of Mathematics* (Even and Ball, 2009) is instructive because it gives a wide variety of examples and analyses.<sup>11</sup>

The last question to be raised in this section concerns the evaluation of the quality of teacher training and its impact on pupils' learning. These are difficult issues and while there is a wealth of literature on pupils' evaluation in mathematics, research on the subject is still in its infancy. Such research is difficult for, as in all evaluations, the tools used are not neutral. They constitute the prism through which training courses are evaluated. They imply assumptions about the knowledge and competencies expected of such training and about the methodologies used to "measure" whether they have been acquired at the end of the training course. If large samples are targeted, data cannot be collected in the actual context in which the profession is exercised but this context can at best be simulated. The first IEA teacher-training survey was devised with this in mind (Tatto et al., 2008).<sup>12</sup> The quest for links between the teachers' knowledge and competencies and their pupils' learning raises the question of discriminating among factors and of understanding mechanisms likely to explain those links, in addition to identifying correlations or statistical implications. This, too, is a relatively new and difficult form of research (Hill et al., 2007) and it may yield interesting results because, as underlined above, great progress was achieved in qualitative research on teaching practices and their determining factors in the last decade.

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11 The case of "Lesson Studies", a device for professional teacher development in Japan which became a focus of attention when the results of the TIMSS international evaluation drew attention to educational practices in that country, is given in the annex.

12 A description is given in the annex.

## 6. Achieving synergy among all

It is evident from all of the above that, to take up the challenge of quality mathematics education for all, synergy must be achieved among a variety of experts such as mathematicians, teachers, teacher trainers and educationists in particular. It is not easy to achieve synergy. Mathematicians have nevertheless been traditionally active in educational issues, at least in certain countries, as borne out, for example by the history of ICMI, which celebrated its first centenary<sup>13</sup> in 2008. The commission itself was established at the fourth international congress of mathematicians in Rome in 1908, and its first president was Felix Klein, the great mathematician. He was the author of the well-known “Elementary Mathematics from an Advanced Standpoint” books written for teachers and designed to close the then existing gaps between university and secondary-school mathematics. During the first hundred years, ICMI was at the interface between mathematics and mathematics education, seeking to strengthen synergy with varying degrees of success (Artigue, 2009). There is still much to be done today. It seems that two challenges, in particular, must be met, namely broader involvement of mathematicians and recognition of their commitment on the one hand and, on the other hand, better collaboration among mathematicians, educationists, teachers and teacher trainers. These challenges are quite difficult to meet in many developing countries, which concomitantly face several difficulties arising from the small number of mathematicians who already face many other responsibilities and demands, the large number of pupils and teachers to be supported and the lack of a tradition similar to the one described above.

It is evident from all of the above that, to take up the challenge of quality mathematics education for all, synergy must be achieved among a variety of experts such as mathematicians, teachers, teacher trainers and educationists in particular.

### 6.1 Broader and better acknowledged involvement of mathematicians

As pointed out earlier, mathematicians in many countries have traditionally been involved in issues relating to primary and secondary education and teacher training, but they have often concentrated on finding and nurturing future mathematical talent. This has been borne out

13 Information is available on the historical website of ICMI <http://www.icmihistory.unito.it/>.

by their involvement in the organization of various mathematical competitions, in particular Mathematical Olympiads. This choice is understandable but, if quality mathematics education for all is to be a success, mathematicians must be more broadly involved, addressing a less specific audience in forms other than the organization of competitions. There are many examples of such initiatives today, which led ICMI to launch a study entitled *Challenging Mathematics In and Beyond the Classroom* (Barbeau and Taylor, 2009). Several examples are given in the annex, but they do not faithfully reflect the wealth and diversity of mathematicians' current activities, which do or can contribute to quality mathematics education for all. That said, it remains that mathematicians' involvement is generally personal, winning little institutional support, and undervalued. For matters to be otherwise, it would be necessary to break with a system in which only the researcher's productivity is valued professionally, as is generally the case, unfortunately. This situation is particularly problematic for young mathematicians who, however, are especially capable of showing pupils that mathematics is a living science.

## 6.2 Better inter-community collaboration

The second challenge is that of better collaboration among the various communities tasked with educational issues, in particular mathematicians, teachers and educationists. From this point of view, in the last few decades, the development and institutionalization of didactics as an academic research field, fuelled by the disillusionment generated by the period of modern mathematics, have modified the traditional balance. In the last decade, in a number of countries, dissatisfaction

Better collaboration among the various communities tasked with educational issues, in particular mathematicians, teachers and educationists.

with mathematics education has resulted in mistrust and even rejection of research ideas which, if not actually implemented in practice, were reflected in a number of curriculum documents. This is particularly true in countries where mathematicians and mathematics education researchers work in separate institutions and hardly work together on teacher training, among other topics. This situation seems to be highly detrimental to mathematics education. It is not, however, inevitable and it is therefore important to raise awareness of successes in that area and to hold them up as a source of inspiration.<sup>14</sup>

<sup>14</sup> Two examples, the case of the French Mathematics Education Research Institutes (IREMs) and of the Park City Mathematics Institute in the United States of America, are given in the annex.

## 7. Organized complementarity between formal and non-formal education

**A**s emphasized above, mathematics taught during basic education must be stimulating, an instruction in living mathematics relating to the world in which the pupils live and to issues faced by humanity today. Reference has been made to achievements that seem to indicate that this is possible, in various ways, depending on choices made and on social and cultural contexts. However, within the usual confines of school organization, in which subjects are often highly compartmentalized, there are limits to what can be done. It is therefore important to afford opportunities for everyone within the school to engage freely in activities that are more open to another temporality and a different sort of didactic management. It is also important to draw on the many non-school learning opportunities open to young people today. This is still a challenge for, although schemes abound, only very few basic education pupils take up such opportunities to date. The fifteenth ICMI study mentioned above gives an idea of the large number of ongoing schemes, which are divided into sixteen different categories, each illustrated by specific examples. As this text is produced for UNESCO, mention should also be made of the UNESCO travelling exhibition “Experiencing Mathematics!” which has travelled round the world three times since 2005. It is now complemented by a virtual exhibition and is generally accompanied by various mathematics events that draw not only school classes but also a very large public audience.<sup>15</sup>

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<sup>15</sup> Detailed information on this exhibition is provided at <http://www.mathex.org/MathExpo/> from which the virtual interactive exhibition can be downloaded in English, French, Portuguese and Spanish.

## 8. Change management and adjustment

To achieve the goal of quality education for all, undeniable progress must be made and many experimental achievements can be used to plan such changes. Nevertheless, it must be acknowledged that attempts at large-scale changes, even when conducted on the basis of preliminary experiments, have generally been disappointing. Change management and adjustment in education systems are particularly problematic. Although the analysis of past experience does not yield a reliable guide to the future, it at least reveals errors that should not be repeated, some of which are mentioned below, with particular emphasis on developing countries.

A severe but useful criticism of the way in which mathematics curricula are often changed in developing countries, on the initiative or with the support of international agencies, is levelled by Bienvenido F. Nebres (Nebres, 2009),<sup>16</sup> who gives his own country, the Philippines, as an example. He describes typical curriculum development as divided into four phases, namely the introduction of a new teaching approach inspired by a Western theory (such as modern mathematics, “back to basics”, problem-solving and constructivism), the production of textbooks and resources based on these approaches, ever successful pilot and small-scale studies in particular contexts and national implementation. Implementation is backed by a teacher training scheme following the “cascade model”, with substantial training at the highest levels but generally reduced to two or three weeks for teachers tasked with implementing the reform in the classroom. The new curriculum sweeps away the old one, the teachers must adapt to it abruptly with minimal training, the results are poor and a few years later, a new curricular project is launched to remedy the situation. He contrasts this caricature of reform with the rounds of curricular reforms in Japan, which cover a 10 to 12 year period and ascribe great importance to adjustments, starting with the routine collection of teachers’ reactions, analyses and overall views, and discussions held at all levels of the education system in order to reflect and decide on the necessary changes. Strategies have been formulated in the Philippines to break with this situation in a series of important reforms implemented in the last decade under the Third Elementary Education Project, from 1998 to

16 For a more comprehensive analysis, see Atweh, Clarkson and Nebres, 2003.

2006, and the Synergeia Foundation.<sup>17</sup> These and several other analyses and efforts throw light on a number of important principles that guide such activities. These principles are unfortunately seldom respected in reforms implemented in either developing or developed countries:

- the importance of the political, economic, social and cultural context – although counterexamples abound, the illusion too often persists that one can “borrow” a device, a form of curricular organization that “works” in another context to improve one’s own, and that by reproducing that form one will score a certain success; successful adaptation, where possible, entails transposition informed by an understanding of the characteristics and processes that make the particular form of organization a success;
- the importance of time frames – in education, experience shows that projects that have a substantial and lasting impact are necessarily projects that require coherent action for at least one decade;
- the importance of thinking changes through changes to the curriculum or to practices, as departures from the existing curriculum or practices and not as a revolution – changes to practices, in particular, must be regarded as dynamic and be guided for a sufficient period of time by models that do not follow the cascade scheme; fieldwork with teachers and the training of resource persons locally are crucial to maintaining the momentum and ensuring that the actual changes are sustained beyond the period of institutional support;
- the importance of breaking with top-down changes and of striking the right balance between institutional drive and the contributions of field players – in other words “top-down” and “bottom-up” changes must be balanced in educational design and management; in that context, non-school communities must be involved in the changes as much as possible;
- the importance of including assessment and adjustments – the effects of an action on an education system are seldom those expected;
- the importance of carefully preparing for and monitoring the effect of changes of scale – pilot experiments are useful but they seldom deliver all of the necessary keys to successful changes of scale.

If these principles are respected, substantial and lasting changes should be achieved and wide swings, to which education systems are unfortunately often prone, should be avoided.

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<sup>17</sup> These strategies and their positive effects are detailed in Nebres, 2009. In the annex, this information is supplemented by that provided by Professor Merle C. Tan, Director of the National Institute for Science and Mathematics Education Development, University of the Philippines. The activities described therein are designed to expand quality mathematics education for all in order to meet the needs described in the document. These efforts are supported by examples of international collaboration, such as participation in the above-mentioned Learner’s Perspective Study.

## 9. The technological challenge

At the beginning of this paper, it was stressed that quality education for all today cannot be achieved without taking technological factors into account. It was stressed mainly that the concept of mathematical literacy should take into account the technological tools that are used in social practices today and, in particular, in basic education, the practices of calculation. The expansion of access to data, means of representation, interaction among representations produced by digital technologies and the way in which technological development influences the development of mathematics itself, in particular because of interaction between the mathematical sciences and computer science, were also mentioned.

Quality education for all today cannot be achieved without taking technological factors into account.

That portrays only part of today's technological challenge and, to supplement that view, emphasis will be laid specifically on changes initiated owing to technological developments in training, collaboration and exchange, access to and production of educational resources. Initially the discussion on the potential of technology-mediated mathematics education was focused on the use of calculators or software designed either for education or for professional use and then converted into educational tools such as algebraic computing software and spreadsheets. This featured in the first ICMI study on the subject, the second edition of which was published by UNESCO in 1992 (Cornu and Ralston, 1992). In basic education, the technology concerned was mainly calculators, spreadsheets, dynamic geometry software and micro worlds such as Logo. As shown by the second ICMI study on the subject (Hoyles and Lagrange, 2009), these technologies have undeniably enriched opportunities for experimentation, visualization and simulation and have modified relations with calculation and geometrical figures. They have brought school mathematics closer to the outside world by making it easier to process more complex data and to handle more realistic problems. However, in spite of their undeniable potential for enhancing the teaching and learning of mathematics and their many positive achievements, they have to date had little effect even in education systems that strongly encourage their use. Recent work on teachers' practices in computer environments is beginning to give insights into this situation, and forms of training properly adapted to teachers' needs are being considered. Nevertheless, the issue of widespread effective use of these technologies in basic mathematics education remains for the moment unresolved.

Recent developments, such as those relating to the growth of collaborative learning tools, the Internet and mobile technologies, have given rise to different opportunities with differing impacts, as evidenced by the option of using technology to support forms of collaborative mathematics learning by students, free online access to a range of resources, new options for organizing distance education and support for the collaborative production and sharing of resources, for the emergence of communities of teachers and researchers and for networking and remote exchanges between pupils and teachers. As shown by the second ICMI study (Hoyles and Lagrange 2009) mentioned above, new learning opportunities are thus opening up, facilitating access to resources and further training, combating isolation, supporting the diffusion of ideas and innovation and breathing life into UNESCO's values of solidarity. This seems to be particularly promising for everyone and for developing countries in particular. It is therefore important to take advantage of these mathematics education opportunities, especially as their integration does not seem to raise the same problems as that of the technologies mentioned earlier since they do not similarly affect practices.

The issue of technology should desirably be linked to that of teaching resources. There cannot be any quality mathematics education for all unless quality resources are produced for pupils and for teachers. The recurrent difficulties encountered in spreading acquired knowledge about teaching and learning and in disseminating innovation call into question both the design of resources and the processes used for their dissemination. The problems posed by resources that are supposed to support changes required to practices but are too different from usual practices to be in the target users' "proximal zone of development" has been highlighted in Section 3. Another problem is that existing resources are often

There cannot be any quality mathematics education for all unless quality resources are produced for pupils and for teachers.

not designed to make it easy for teachers to adapt them as necessary to their particular teaching context or those teachers are not trained to make such adaptations. Available knowledge of teachers' documentary practices is simply not sufficient to provide satisfactory guidance on teacher training. However, emerging research in this field (Gueudet and Trouche, 2009) shows that rapid developments have been triggered by technological change, in particular the burgeoning of accessible online resources and the support and encouragement for collaborative work.<sup>18</sup> There is certainly new scope for designing and disseminating resources and, without any doubt, for also setting new requirements for teacher education.

<sup>18</sup> The case of the Sesamath association, illustrative of developments observed from this standpoint, is outlined in the annex.



## 10. Collaboration

The challenge of quality mathematics education for all will not be met unless collaboration is strengthened. Collaboration has traditionally been viewed in terms of North-South cooperation. Such collaboration is indeed essential, and projects on mathematics and mathematics education already abound. However the importance of regional collaboration on mathematics education merits recognition on a wider scale. As already emphasized, mathematics education is rooted in contexts and cultures that must necessarily be taken into account in order to secure educational improvements. Regional collaboration plays a crucial role in that regard. ICMI, for example, has gradually established regional bodies. The first, the Inter-American Committee on Mathematics Education (IACME), was established in Latin America in 1962 and was followed, in 1976, by the establishment in South East Asia of the South East Asia Conference on Mathematics Education (SEACME), which was subsequently extended and renamed East Asia Regional Conference on Mathematics Education (EARCOME). The last two bodies established are the Africa Regional Congress of ICMI on Mathematical Education (AFRICME) for English-speaking Africa, and the Francophone Mathematics Space (EMF), a new language-based mechanism enabling ICMI to capitalize on cultural proximities and to promote solidarity between centres and peripheries. Reflection on the best ways and means of organizing regional and international solidarity is analysed in detail in the DCSG 2009 document on Africa.<sup>19</sup> Emphasis is laid on complementarity between various types of collaboration and on the importance of forming networks and of providing African students and researchers with the means of finding sufficient resources regionally in an endeavour to reduce the brain drain. The document draws attention, as noted above, to the new technology-mediated opportunities for achieving such goals.

<sup>19</sup> See, too, the final report of the Oslo conference mentioned above (<http://www.dnva.no/c26889/binfil/download.php?tid=27685>), which also highlights the interest shown by regional cooperative bodies, notably, in regard to mathematics, the Latin American and Caribbean Mathematical Union (UMALCA) and the African Mathematics Millennium Science Initiative (AMMSI), and the role played by UNESCO-supported bodies such as the International Centre for Theoretical Physics (ICTP) and the International Centre for Pure and Applied Mathematics (ICPAM), whose ongoing action in Cambodia is outlined in the annex.

## 11. The diversity challenge

**D**ifferent forms of diversity – socio-economic, cultural, linguistic and gender diversities, among others – are a challenge to quality mathematics education for all. Emphasis will be laid below on language issues and gender issues, without underestimating the many forms that diversity takes or the effects of their interaction. The importance to be ascribed to contextual and cultural differences has been stressed time and again in this document. The field of ethno mathematics (D'Ambrosio, 2008) has contributed substantially to raising awareness of cultural diversity issues and their educational implications for mathematics. It seems important to stress that the issues involved here are delicate to handle. Attention to cultural diversity must not lead to isolation and it is important to help pupils to see how mathematics, owing to its universal value, can play a special role, together with the other sciences, in the rapprochement of peoples and cultures and in enabling mutual understanding and collaboration.

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### 11.1 Language issues

The problem here concerns the provision of mathematics education in a language that is not the pupils' mother tongue,<sup>20</sup> a problem that arises in a number of developing countries in which the language of instruction is the language of the colonist and in which many local languages often coexist.

Owing to migration, however, this problem is far from confined to developing countries. In such a situation, it is more difficult to express and work on ideas and emerging constructions before more standardized forms of mathematical expression are accessible and usable. This difficulty has a particular impact on the first few years at school, and it is important to bring it to the notice of education systems and teachers. It is, however, a problematic issue for, although it is important to take linguistic diversity into account, this must not be done in a manner that could hinder social integration. There is now a substantial body of work to fuel reflection and decision-making

<sup>20</sup> UNESCO has already stated its position on these issues (see UNESCO, 1953; UNESCO, 2003).

on the subject (see, for instance, Secada, 1992; Adler, 2001; Setati, 2005; and Moschkovich, 2007 and 2009). For this reason, a study was launched recently on the topic by ICMI.<sup>21</sup> Furthermore, linguistic diversity should not be seen only as a difficulty, but also as a potential source of enrichment for teaching and learning.

## 11.2 Gender issues

Gender issues arise in mathematics as early as the compulsory education stage, owing to the differences in access to basic education afforded to girls and boys in a number of countries. Where there is equality of access to education, one might think that gender issues have been resolved, especially now that girls are more successful academically than boys and that the mathematics performance gap is being closed, in a number of countries at least. This is, however, a mistaken view because inequalities in performance still exist and are even emerging again in countries where they had been resolved; moreover gender issues cannot be approached only in terms of accessibility and success during compulsory education. The way in which pupils experience mathematics during basic education is crucial to their future. Many studies show that girls and boys are treated differently in class and that the activities set for pupils, the conduct of those activities, the roles assigned to girls and boys and the way in which their work is evaluated contribute to such differentiation. This increases the significance of cultural and social stereotypes, with the result that girls who achieve the same results as boys are less confident than boys about their ability to pursue studies in mathematics and tend to exclude mathematics from their prospective career.

The existing difficulties are well documented by research and various syntheses are available. These resources include reports on many demonstrably effective activities (see, for instance, Hanna, 1996; Leder, Forgasz and Solar, C, 1997; Corbett, Hill and St. Rose, 2008; Leder and Forgasz, 2008). It is important to raise teachers' awareness of these issues during initial and in-service training.<sup>22</sup> It is also important for the activities carried out to be given institutional support and a high profile.

21 The discussion document for this study (ICMI 21) is available on the ICMI website.

22 See, for instance, the website of the "Femmes et Maths" association, which contains many activities for both pupils and teachers ([www.femmes-et-maths.fr](http://www.femmes-et-maths.fr)).

## 12. The challenge of research

The challenges of quality mathematics education for all cannot be taken up without developing new knowledge through research. In the last few decades, mathematics education research has surged. It initially improved understanding of pupils' learning processes, the difficulties and obstacles to be overcome in learning rudimentary concepts taught during basic education, the modes of reasoning and proof, and the representations and languages used to gain access to mathematical objects. It also improved insights into the functioning of didactic systems, the transposition of mathematical knowledge which occurs in those systems and the interaction among the various players and their effects. It explored the potential of technology-mediated learning and ways in which such potential may or may not be harnessed in actual classroom practice.

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Research has begun to focus more recently on teachers, their convictions and beliefs about mathematics and mathematics education, their knowledge and competencies, and ways and means of developing the latter. It has sought to grasp the complexity of teaching practices, their determining factors and the processes involved in their development. It has examined teacher education and its effects and has sought to improve understanding of the cultural facets of teaching and learning.

A substantial body of knowledge has thus been formed, and its proportions can be gauged from the work entailed regularly in its reorganization and synthesis (see, for instance, Bishop et al., 1996; Bishop et al., 2003; Gutiérrez and Boero, 2006; Lester, 2007; and the ICMI studies mentioned above). However, that knowledge still has limited potential to drive the necessary changes in varied and often difficult contexts in which very few means of action are open. Nor does it suffice to anticipate the possible effects of the educational choices made and to address change-of-scale issues. In addition to fundamental research aimed at identifying and understanding phenomena, it is essential to conduct research that is more focused on didactic action, with contexts and changes of scale as core issues. From this point of view, the renewed interest in design-based research must, doubtless, be encouraged (Design-Based Research Collaborative, 2003).

Nevertheless, it must be stressed that educational issues are so complex that research in mathematics education on didactics can point to only part of the approach and so cannot guide

the necessary developments on its own. It is important to promote interaction with other fields of research on teaching and learning in general and it is equally important to be able to analyse the present and to think about the future from a historical standpoint, whether in terms of the history of mathematics or the history of education.

# Summary


 ctually providing quality mathematics education for all during basic education is an ambitious challenge whose success, apart from access to basic education, depends primarily on countries' capacity to train and retain a sufficient number of qualified teachers. Training qualified teachers is synonymous with training teachers to teach in a stimulating manner that portrays mathematics as a science concomitantly rooted in history and living in the present, in tune with the world and able to contribute to the solution of world problems and to bring people closer together, owing to its universal values. Ensuring that mathematics is perceived as a science accessible to all and able to provide anyone with unexpected aids to understanding and action is yet another challenge. It entails regarding mathematics education as connected with the teaching of other disciplines, especially scientific disciplines, and not as an isolated subject. Such a project will only succeed if everyone – mathematicians, teachers, teacher educators, educationists, specialists, teachers of other disciplines and decision-makers – works together to take up this challenge and if regional and international cooperation and solidarity are strengthened. The potential for achieving this does exist, even in the most disadvantaged areas. Furthermore, many activities are carried out in a wide variety of economic, social and cultural contexts that can all inform policies. It is necessary to analyse those activities, measure their effects and study the conditions and processes that determine their potential and limits. It is also necessary to know that solutions to educational problems cannot be found without taking contextual factors seriously into account. Owing to technological advances today, new approaches are being taken to this challenge and it is crucial that they be turned to good account. However, no action will lead to substantial and sustainable results if the importance of quality mathematics and science education for all is not acknowledged as a policy priority, together with all related implications, particularly for teachers' initial and in-service training, status and working conditions.

Ensuring that mathematics is perceived as a science accessible to all and able to provide anyone with unexpected aids to understanding and action is yet another challenge.

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# Annexes

In the following annexes, we illustrate the different sections of this document with some examples. Due to the limited time available for the preparation of this text, what is proposed is very reduced and does not pretend in any way to cover the diversity of existing achievements that may be sources of inspiration. We tried however to select achievements that are diverse in terms of issues addressed, choices made and contexts. To move towards quality mathematics education for all, it seems important now to find ways to more systematically identify existing achievements, carry out analyses with the level of depth necessary for drawing lessons from them, and then make these analyses widely accessible.

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Annex 1	Connections between mathematics and science education in the German SINUS programmes
Annex 2	Forty years of working on mathematics education, seeing mathematics as a human activity for all – The Freudenthal Institute
Annex 3	Problems and challenges in mathematics education: the Philippines Context
Annex 4	The professional development of teachers in Japan – the concept of Lesson Study
Annex 5	The professional development of mathematics teachers in Brazil: structural issues, initiatives and hopes
Annex 6	Systematizing knowledge about mathematics teacher education – The IEA Teacher Education and Development Study in Mathematics (TEDS-M)
Annex 7	Research on mathematics teacher education in South Africa and Southern Africa
Annex 8	Promoting excellence in mathematics teaching The National Centre for Excellence in the Teaching of Mathematics (NCETM)
Annex 9	Experiencing Mathematics! an international travelling exhibition
Annex 10	Challenging Mathematics – Mathematics Houses in Iran

- Annex 11 Collaboration between mathematicians, teachers and researchers in mathematics education – the case of the IREM network
- Annex 12 The emergence of a teacher community – the case of Sesamath
- Annex 13 Fostering interactions and collaborations “Teacher Education Around the World: Bridging Policy and Practice,” a component of the IAS/Park City Mathematics Institute for Advanced Study, Einstein Drive, Princeton, New Jersey
- Annex 14 Rebuilding a mathematical community in Cambodia
- Annex 15 List of participants at the Experts' Meeting

# Annex I. Connections between mathematics and science education in the German SINUS programmes

(With the collaboration of Volker Ulm<sup>1</sup>)

In the document, we have insisted on the necessity of improving the relationships between mathematics and science education. The SINUS programmes in Europe provide an insightful illustration of what can be achieved in that area. An important characteristic is also the up-scaling process that is currently organized in the framework of the Fibonacci project beginning in 2010.

In recent years three programmes have had remarkable influence on mathematics and science education in primary and secondary schools in Germany: SINUS (1998 – 2003), SINUS-Transfer (since 2003) and SINUS Primary School (since 2004). Meanwhile about 3000 schools have been involved in this development process of the educational system. Each school concentrates on some areas of activity like “developing a task culture”, “cumulative learning”, “autonomous learning”, “promoting girls and boys” or “working in a scientific manner”. The central website is: <http://sinus-transfer.eu>

In these SINUS programmes mathematics and science education cross-fertilize on different levels.

## I. **The first level is an obvious one: it is the level of tasks and topics.**

The development of a task culture in mathematics education intended in the SINUS programmes includes the emphasis of problems which arise in realistic situations – especially in nature. They require understanding for nature and sciences as well as competences of mathematical modelling. Thus, mathematics education needs sciences. Vice versa, quantitative working on problems in sciences requires mathematics. So students get aware that mathematics and sciences provide

<sup>1</sup> Volker Ulm is Professor at the University of Augsburg, in Germany, Chair of Didactics of Mathematics.

different and complementary perspectives on the same subject: on nature. It is one objective of the SINUS programmes to make students develop this insight and to make them aware of the potential of the interplay of mathematics and sciences. For that a special module “Experiencing subject boundaries and interdisciplinary approaches” has been established within SINUS.

## **2. The second level is a more sophisticated one: it is the level of didactic concepts and methodology.**

On the one hand the experimental approach to problems in sciences has been transferred to mathematics education. According to this concept students are given mathematical situations and are invited to explore them in an experimental manner. In this process software, e. g. for dynamic geometry, has proven to be a useful tool. In an idealized way experimental teaching and learning scenarios can be described by the following scheme:

- Confrontation with a mathematical phenomenon
- Exploration of the problem field, e. g. with use of ICT as a tool
- Structuring of the findings
- Noting down the results
- Presentation and discussion in class

Especially in the phase of exploring the problem field the computer may be compared with an experimental apparatus in natural sciences.

This concept of experimental mathematics helps to implement general didactic and pedagogical ideas like inquiry-based, autonomous and cooperative learning in everyday mathematics lessons.

On the other hand didactic concepts and methodologies which had originally been developed for mathematics education have been transferred to science education. For example, many schools in the SINUS programmes focused on the opening of tasks, on self-responsible and cooperative learning and on the securing of basic knowledge in mathematics. Teachers developed their way of posing questions, of structuring lessons and of dealing with mathematics. There is a shift from teacher-centred instruction to students' work in learning environments. Since these developments work on the level of general attitudes towards teaching and learning and since many mathematics teachers in Germany also teach sciences these processes strongly influence science education. So didactic concepts mainly developed for mathematics education get successfully applied to science education. This works well, since the structural problems and

the necessities of development are quite similar in mathematics and science education and since the solutions and concepts of the SINUS programmes are quite general and transferable.

A perspective: The systemic school development processes of SINUS in Germany will be extended to the European level in the framework of the programme "Fibonacci". From the beginning of 2010 the ideas of SINUS and the corresponding French science project "Pollen" will be spread out and transferred to the educational systems of at least 21 European countries.

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## Annex 2. Forty years of working on mathematics education, seeing mathematics as a human activity for all – The Freudenthal Institute

(With the collaboration of Marja van den Heuvel-Panhuizen<sup>2</sup>)

Since the late 1960's the Freudenthal Institute of Utrecht University has worked on the improvement of mathematics education. The inspiration for this work lays in the profound belief that the global community of mathematics education developers and researchers – including the Freudenthal Institute staff – has the responsibility for providing students of all ages – to begin with the very young children in pre-school settings – with the best possible learning environments for developing mathematical skills and concepts. The relevance of learning mathematics is not only that it is necessary for supporting life on earth with sustainable technology and economy, but also that acquiring mathematical competence is of high personal value. Mathematics is, together with reading and writing, one of the key human competences through which people can express themselves as human beings and can understand the world around them. Freudenthal's idea of “mathematics as a human activity” and his quintessential and ambitious goal of “mathematics for all” have always been the Institute's guiding principles for researching and developing mathematics education. This resulted in a domain-specific didactical theory that nowadays is called “Realistic Mathematics Education” for which Treffers and his colleagues of the former “Wiskobas” group laid the foundation.

“Mathematics as a human activity” means the opposite of the traditional transition approach to teaching mathematics. Instead of teaching students ready-made mathematics, they should be given the opportunity to develop mathematics by a process of guided-reinvention. This means that the teacher plays a strong pro-active role by creating a stimulating and supportive learning environment. “Mathematics for all” means making mathematics accessible for students at all intellectual levels. This requires a deep awareness of mathematics as more than just the most abstract way of thinking. Even students with lower potential in learning mathematics can use mathematics to solve problems by using informal context-connected strategies. The

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2 Marja van den Heuvel-Panhuizen is Professor of mathematics education at the Freudenthal Institute for Mathematics and Science Education, at Utrecht University, Netherlands.

heterogeneity of students poses a real challenge to education. "Mathematics for all" also includes the high-ability students.

The work at the Freudenthal Institute characterizes a synthesis of theoretical expertise, empirical knowledge and practice-based experience in the area of learning and teaching mathematics. A multi-disciplinary approach in connection with a strong intertwinement of theoretical and practical perspectives can be found in the design and research methods, as well as in the qualities of the staff involved. Many Freudenthal Institute staff members are researchers as well as curriculum and software designers or are still active as teacher educators or teacher advisors. Moreover, in many projects, staff members collaborate with mathematics teachers and others from school practice. These projects cover a wide scope of educational settings ranging from pre-school education, primary school education, general and vocational secondary school education and higher education. They even include outsidetextbook settings where learning takes place. Essential is that the Freudenthal Institute has chosen for an integrated approach in which research and development, as well as teacher education and implementation through professional development of in-service teachers are closely connected. Moreover, the Freudenthal Institute has always had a good relationship with textbook authors who were free to use the Institute's ideas for teaching mathematics and who, in turn, contributed to the reform movement in mathematics education which has come into being in the last decennia.

- 1 In 2006 the previous Freudenthal Institute which had its focus only on mathematics education merged with the groups for physics education, chemistry education and biology education which resulted in the "Freudenthal Institute for Science and Mathematics Education" as part of the Faculty of Science of Utrecht University. The present text discusses the mathematics education department of this new Freudenthal Institute.
- 2 The ultimate goal of the work at the Freudenthal Institute is the enhancement of the quality of learning and teaching mathematics. The quality of learning signifies that the subject matter knowledge and skills that students have to learn should be meaningful. This means that students have to acquire relevant knowledge and skills, both for the present and for future studies and employment. Another aspect that indicates the quality of learning is whether that learning is effective in terms of its results. Important is also that the learning enables transfer and that students can apply the acquired knowledge and skills to new problems and new situations in and outside school. The quality of education implies having adequate instructional settings, didactical models and tools available in order to achieve a high output in terms of students' competences and attitudes in mathematics.

Last but not least, the quality of teachers is an important factor in mathematics education, both in realizing education and in innovating education. The former is more related to educating prospective teachers and the latter to counselling experienced teachers, however both should be considered in close connection to each other.

Although the reform of mathematics education was quite successful in the Netherlands in terms of results in international comparisons of students achievements (TIMSS and PISA), it was quite remarkable that the process of renewal took place with almost no direct intervention of the Dutch government. Instead, it was the community of mathematics education developers and researchers, mathematics educators and school advisors, textbook authors, and school inspectors who supported the reform and who all made this process happen together with the teachers. If the government played a part in this process, then it was that they funded the setting and maintenance of the infrastructure in mathematics education. This gave teachers and all the others involved in the reform process conferences to meet and websites and journals to share ideas, experiences and materials.

Creating an opportunity for the mathematics education community to work on their own development did not only bring in ownership, but also resulted in a positive cost-benefit ratio. Despite high yields in terms of the students' achievements, for many years the Dutch government made very low expenditures available for education compared to the governments in other countries. This self-contained process of educational reform is not without dangers. For example, the reform process did not generate a national system for professional development of in-service teachers. With respect to this, the TIMSS 2007 report came with hard facts. The professional development in the Netherlands, in contrast to many other countries, ended up at a very bottom position. Not having such a system for professional development is a serious threat of the quality of education. Especially in primary school, where teachers are all subject teachers, it is an absolute necessity that teachers take refresher courses. This is particularly true with respect to the use of new technology in mathematics education.

More than any other development, Information and Communication Technology (ICT) has proven to make a true contribution to the further development of mathematics education. In fact, ICT lies at the heart of mathematics teaching because it gives didactical models such as the number line and the fraction bar a new dynamic life. Moreover, ICT offers students possibilities for modelling problem situations by themselves and creates an environment in which students are encouraged to ask themselves and others questions, to test ideas and find proofs for their conjectures. These activities are what 'doing mathematics' is truly about. They give students a more active role in learning mathematics. This is especially the case where (mini-) games are brought into action in

mathematics education. Software with game characteristics has powerful opportunities to change students' attitudes to mathematics and can bring in rehearsal in a natural way which is crucial for raising the students' mastery level of basic mathematical skills. Finally, internet and mobile technology open a new avenue for distance learning and assessment, and can be a powerful tool to make the latest didactical developments accessible for teachers. Therefore, at the Freudenthal Institute, ICT is a key theme in researching and developing mathematics education. Using ICT to overcome geographical barriers opens a whole novel range of possibilities to realize the idea of mathematics for all.

- 3 The work of the Freudenthal Institute did not escape international notice. Many colleagues from abroad showed interest in the work of the Institute. Often this led to joint projects and collaboration. Since Freudenthal's time, the Institute has been an open research community that collaborates with researchers from various countries and that has a steady stream of international visitors. Except sharing ideas and experiences and jointly carrying out research or development projects, this collaboration also implies that the Freudenthal Institute makes available instructional materials, such as longitudinal learning-teaching trajectories, models of lesson series and ICT-based mini-games, to be used in other countries. Of course, this is not a matter of translation and distribution. Teaching materials developed for Dutch schools cannot be just simply dropped in another country. Adaptation is necessary and this requires a careful process in which the country's culture-specific approach to education is taken into account. This is also the case in the following examples of "Realistic Mathematics Education in transit". Mathematics in Context was one of the major international projects of the Freudenthal Institute that was carried out with the Wisconsin Center for Education Research (WCER) of the University of Wisconsin-Madison in the USA. In this project the Freudenthal Institute developed the draft version of a mathematics textbook series for the American middle grades. This textbook series got its final form through a process of piloting and revision in which the researchers from Madison took the lead.

For more information see:

[http://info.eb.com/html/print\\_math\\_in\\_context.html](http://info.eb.com/html/print_math_in_context.html);

<http://www.project2061.org/publications/textbook/mgmth/report/2context/info.htm>;

<http://www.marketwire.com/press-release/Encyclopaedia-Britannica-Inc-985121.html>.

The TAL project which the Freudenthal Institute carried out commissioned by the Dutch Ministry of Education was aimed at developing longitudinal learning-teaching trajectories for mathematics education in primary school in order to give teachers an overview of how the learning of mathematics proceeds over the primary grades and to bring coherence in the curriculum. Because several countries attached significance to these trajectories the materials were translated into English. This resulted in new projects.

Together with the University of Cape Town (UCT) and the Cape Peninsula University of Technology (CAPUT), the Count One Count All (COCA) project was started (<http://www.fi.uu.nl/coca/>). One of the goals of this project—which was funded by the South Africa-Netherlands Research Programme on Alternatives in Development (SANPAD)—was to provide South African teachers in the foundation phase of primary school with a document that describes the learning pathway for number. The development of this learning pathway was based on TAL and is connected to the South African National Curriculum Statement.

Furthermore, an educational publisher in Mexico is working on a Spanish version of TAL which, among other things, is already used for professional development of teachers in Argentina ([http://www.fi.uu.nl/nl/Poster\\_TAL-Alta-final.pdf](http://www.fi.uu.nl/nl/Poster_TAL-Alta-final.pdf)).

Another example of “Realistic Mathematics Education in transit” is worked out at the Development Institute of Pendidikan Matematika Realistik Indonesia (<http://www.pMRI.or.id/>). The main mission of IP-PMRI is to improve the quality of mathematics education in Indonesia by implementing an Indonesian version of Realistic Mathematics Education. In the DO-PMRI project, IP-PMRI works together with the Freudenthal Institute, APS, and the Indonesian Ministry of Education to enhance the quality of mathematics teachers in a participative way, providing them with teaching tools and methodologies that give Indonesian students better chances to appreciate and understand mathematics.

A last example of a project that goes beyond the boundaries of the Netherlands is the Th!nklets project in which mini-games are developed for primary and secondary education. These mini-games can be downloaded for free (<http://www.fi.uu.nl/thinklets/>) and are very popular, especially among children aged 8-12. Presently the Freudenthal Institute is working on making some of these mini-games suitable for playing on the XO laptop.

For more information on the work of the Freudenthal Institute, see <http://www.fi.uu.nl/nl/brochureFlsme.pdf>.

## Annex 3. Problems and Challenges in Mathematics Education: The Philippines Context

(With the collaboration of Merle C. Tan<sup>3</sup>)

The case of the Philippines has been mentioned in the document (Section 8) and important projects for mathematics education in that country evoked. In this annex, we complement this information by giving more information about the Philippines context and presenting current efforts made in order to cope with the challenges of quality mathematics education for all, moving towards more stimulating teaching practices.

The Philippines is proud to have top winners in prestigious international math competitions. Many Filipino scientists, engineers, and mathematicians have also made their marks even abroad. However, it is sad to note that in general, Filipino students' performance in international and national mathematics assessment studies is below par compared with neighboring countries. A number of reasons have been used to account for this situation.

The ten year basic education program is blamed for the overloaded curriculum. Topics and skills covered over 12 years in other countries are introduced to students in ten years only. Mastery of content is sacrificed over wide coverage. In addition, the same curriculum is used across the country despite the large dropout rates which means that the curriculum does not address the needs of students from different communities. This explains why the scientific literacy and numeracy level of Filipino students is low based on a UP NISMED study.

There is a shortage of qualified math teachers especially at the elementary level where teachers are trained to be generalists. Also, the most experienced math teachers in the best public and private schools have left or are leaving the country. Moreover, those who have specialization in math in secondary schools chose to change career path; they take graduate courses in research or administration and supervision.

Math classrooms are still teacher-centered. To finish the budget of work for a particular period and because of large classes, teachers tend to be transmissive in their approach to teaching rather

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than allowing students to engage in meaningful and challenging practical work activities. Teachers are dependent on textbooks but are not able to detect misconceptions. While research results say that children learn better when taught using their mother tongue, Mathematics is taught in English even in Grade 1. Difficulty in understanding concepts has been traced to difficulty in understanding the English, a language foreign to many students in different communities.

To have Filipino learners who are critical thinkers and are able to use their knowledge in mathematics for generating and communicating new ideas and in making wise decisions to uplift their quality of life, as well as contribute to the creation of a just and humane society, the Philippine government has ventured into a reform program for basic education. Some of the reforms include development of a spiral and integrated mathematics curriculum; more emphasis on practical work based and open-ended approach to teaching, development of standards for effective math teachers, and implementation of a progression scheme to teacher professional development. Preservice curriculum has been revised to increase the number of units in content and pedagogy. Furthermore, community participation in education governance and accountability has been institutionalized, emphasis on teaching competencies associated with child learning rather than accumulation of credentials in preparation, hiring, and supervision has been given attention, and education facilities using cost effective and appropriate technologies has been modernized, among others.

The UP National Institute for Science and Mathematics Education Development as the national center for research and innovations is taking the lead in raising the quality of mathematics education through its participation in the Learner' Perspective Study already mentioned in the document to determine how student learn, promoting varied forms of assessment, and developing student materials aided by technology. These will help move our teachers and students from the "I know" to "I know how to know" mentality so that so that they become critical thinkers and productive citizens of the country.

## Annex 4. The professional development of teachers in Japan – the concept of Lesson Study

The concept of collective study of a lesson, or *jugyokenkyu* in Japanese, disseminated under the name of lesson study, denotes a longstanding model of teacher professional development in Japan to which attention was drawn by the TIMSS international study. Lesson study is a special form of study within the school, or *konaikenshu*, considered to be an integral part of a teacher's work.

The objective is the collective development by a group of teachers of a lesson with general and specific objectives carefully defined from a thorough study of curricular documents, and taking into account the long-term objectives of students' learning. The preparation leads to develop a detailed design for the planned lesson, including predictions about the behavior of students and their possible learning trajectories, and to specify the points on which the observation of the implementation of the lesson will especially focus. The lesson is then implemented by one teacher of the group, the other teachers being present as observers and intervening only exceptionally. The implementation itself is followed by a phase of evaluation and revision, including one or more meetings. The teacher who conducted the lesson presents his impressions and thoughts, the other teachers their observations and reflections, and the deep mathematical and didactical discussion then taking place can lead to a revision of the initial project, which will in turn be tested. The process is thus based on a precise structure whose various components are precisely described in the literature (see for instance Stigler & Hiebert, 1999).

The observation of this device attracted the attention of researchers on the quality of the mathematical and didactical discussions and work it generated among group members. It also drew attention to the role it played in the Japanese context by supporting curriculum changes, evolution of practices, integration of new teachers, and helping them develop their professional competencies.

The system has thus become an object of study, researchers trying to study more precisely its functioning, identify patterns and variability in its implementation, understand the processes that make it an apparently effective means for the improvement of education in the Japanese context, and what is replicable in other contexts and under what conditions (see Fernandez &



Yoshida, 2004, Isoda *et al.*, 2007 for instance). This is all the more important as this system has now migrated beyond the Japanese context, being in particular imported to the U.S. where it is growing rapidly.

The article (Lewis, Perry & Murata, 2006) focuses on these issues. It proposes different assumptions about how this system can contribute to improving mathematics education: the process of progressive refinement of the lessons, the deepening of teachers' knowledge (mathematics knowledge, knowledge of teaching, ability to observe students, linking the daily activity with the long-term goals of education), teacher motivation and commitment in a community (motivation for change, relationships with colleagues who may provide support, sense of responsibility towards the community), produced resources (lesson plans developed that focus on the students' thinking modes, tools that support collaborative learning over the progress of the system). But the authors also emphasize that in order to have a real understanding of the mechanisms involved, research is needed that is only emerging. Three priorities are identified: enlarging the database related to the system, both in Japan and in countries where it is implemented, particularly in the United States, understanding the mechanisms by which this system affects education, testing cycles of design - implementation - evaluation - improvement, and analysing their effects. According to the authors, such priorities, however, go against a vision of research that, in the U.S. in particular, values above all new ideas at the expense of refinement of existing approaches and of the study of innovations initiated by practitioners. It requires recognizing the relevance of what they call a «local proof route» by which innovations based on local initiatives can lead to substantial and large scale improvements, through the efforts of researchers for explaining and supporting the development and systematically testing these innovations.

The system of lesson studies and the discussions and studies that it motivates thus illustrate several ideas that were developed in the document: the importance of thinking in terms partially renewed the evolution of practices and modes of teachers' professional development, focusing on the professional tasks related to teaching, first and foremost the design and implementation of teaching sequences; the importance for the professional development of teachers of collaborative work in communities of practice; the importance for research to be open to innovations coming from the terrain, and also the need for research to understand the underlying mechanisms and evaluate their effects; and the necessary vigilance with respect to hasty transpositions of what works or seems to work in a given context, paying often attention only to the most superficial features.

Finally, we would like to emphasize that the ICMI Study mentioned in the document (Even & Ball, 2009) shows many examples of teacher training practices which, without necessarily reaching the sophistication and scale of lesson studies, also adhere to such principles. These strategies are fostered by the technological evolution which, allowing collaborative work on videos, promotes a remote access to practices and to forms of training taking better care of the real needs of the practice.

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## Annex 5. The Professional Development of Mathematics Teachers in Brazil: structural issues, initiatives and hopes

(With the collaboration of Yuriko Yamamoto Baldin<sup>4</sup>)

This annex reports on some initiatives concerning the professional development of mathematics teachers in Brazil, from the perspective of the education courses for their preparation and the late efforts made by the Ministry of Education to improve the knowledge and the practice of school teachers. This text intends to give an overview of the state of some recent projects on Continuing Education of Mathematics Teachers in Brazil. It is introduced by a short introduction to the education system of teachers of Brazilian basic schools, the main problems faced by the instruction of Mathematics in elementary levels, and the recent initiatives taken by the Ministry of Education and Universities, by means of collaborative projects.

**The Brazilian Basic School System:** The basic school system in Brazil is divided in by two parts: Fundamental and Secondary levels. Fundamental school goes from 1st to 9th grade, starting at 6 years of age. Kindergarten is not mandatory in Brasil, but the late educational policy stresses strongly the importance of the investment of government in this direction. The Secondary level corresponds to 10th to 12th grade, and it is regulated by the Secretary of Education of each State of Federation. Kindergarten and the first five grades of fundamental level are the responsibility of each town/city, with expectations that in few years they will be responsible for the whole cycle of the fundamental school system. At the moment, the last four years of fundamental level may be under the responsibility of local State, being in any case run differently from of the five first school years. The National Curriculum Standard Documents (Parâmetros Curriculares Nacionais) are released by the Ministry of Education and are, in general, incorporated into each State Educational policy.

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4 Yuriko Yamamoto Baldin is Associate Professor in the Mathematics Department of the Federal University of São Carlos (Brazil), and the ICMI Representative for Brazil.

In the publication cited in reference (1) it is observed that:

“the disruption of educational policy in the middle of elementary schooling is felt as one of the causes of the difficulties to follow a sensible pedagogical planning for mathematics education at this level. Besides being a political issue, the serious problem is that the education of teachers suffers also a discontinuity. The courses for preparation of teachers of first 5 years are deficient in mathematical content as well as in the teaching methodologies for this discipline. On the other hand, the courses that prepare teachers for 6th to 9th grades usually do not focus on the actual learning phase of the students that will face new level and enter into a new system, so that the transition from elementary to middle years of fundamental schools are troublesome.”

The courses that would prepare teachers for the last years of the fundamental level and for the secondary level are undergraduate courses at Universities, and they are those educating for specific discipline. The most of the teachers of first years of the elementary school have, when they have studied at university level, graduated in Pedagogy with little or no mathematics content. Therefore, the lack of consistent formation of the most of the elementary school teachers as well as the unawareness of the secondary school teachers about the transition between levels are the main problems of mathematics teacher education in Brazil. Moreover, due to the large area of the territory together with the economic and cultural differences among geographic regions, the social dynamics of the country implies other serious issues, so that it demands actions from diverse sectors of the society besides the initiatives of the central government. One real problem is that in many regions there are school teachers without formal education, they are recruited in a temporary basis to occupy the teaching positions needed in local schools. The geographic area of Brazil is over 8.5 million km<sup>2</sup>; more than 46 million students were enrolled in over 165.8 thousand public schools at basic education level in 2007, according to the Ministry of Education Data Base ([//www.inep.gov.br](http://www.inep.gov.br)).

Some recent initiatives concerning the professional development of Mathematics teachers: The existing formal presential Courses in Universities have been shown to be insufficient to attend the enormous number of enrolled students in fundamental schools. In order to attend as fast as possible the large demand for qualified teachers in basic education, the government has implemented, starting in 2007, the Distance Education System (UAB = Open University of Brazil), using the technological infrastructure of academic and technical universities around the country. Specifically for preparation of teachers in Mathematics for middle fundamental level (6th to 9th grades) and secondary level (10th to 12th grades) there are by now 27 institutions offering undergraduate courses based on the principles of distance-education, with follow-ups through tutoring and on-line didactical materials. This system is aimed primarily to minimize the shortage of teachers at the basic schools. The impact of this system on the quality of education should be analyzed in near future, as the graduates enter the job market.

For Continuing Education of Teachers, there has been a number of initiatives, some started as outreach projects by universities and research groups supported by governmental financing agencies like CAPES ([//www.capes.gov.br](http://www.capes.gov.br)) of the Ministry of Education and others. For instance, some of the Pro-Ciências/CAPES Project, carried out in the mid 1990's, aimed at updating the formation of secondary school teachers in regard of innovative concepts such as the inter and intradisciplinarity of Mathematics and Sciences and the use of Technology in Teaching and Learning Methodologies, but they had isolated impacts regionally and in small scale. More recently, the Ministry of Education has established a so called Rede Nacional de Formação Continuada de Professores, literally the National Network for Continuing Formation of Teachers, which main goal is to provide a nationwide organized system of courses (one of which in Mathematics content) to the teachers of elementary level, that would support the achievement of both knowledge and teaching methodologies. As special project for the mathematics literacy of elementary school teachers, the Pro-letramento project is being carried out throughout the country by the universities and researchers of "Rede", since 2006. This initiative has reached so far about 50.000 teachers of elementary level, who have undergone a 6 month period of instruction, with many more still continuing the studies. An interesting report (2) has analyzed the possible relationship between the Pro-letramento in Mathematics and the improvement in mathematics scores by the students of elementary schools, through a comparative study of the data from the National Assessment of Basic Education of the Ministry of Education (SAEB) in years 2005 and 2007, that is, before and after the start of the project. The study has found a remarkable change in the differences of scores in the regions in which the project was executed, mainly in the north and northern part of the country, traditionally scoring worst in the assessments. The analysis will be carried out in long term as the projects goes on.

Another important initiative supported by the government is the special program attached to the project of the Brazilian Olimpiad in Mathematics for Public Schools (OBMEP), executed by IMPA (Associação Nacional de Matemática Pura e Aplicada), an internationally renowned research institute, and SBM (Brazilian Society of Mathematics). The OBMEP, started in 2005, is aimed at the 6th to 12th graders of public schools, with more than 19 million of participants in 2009, from 43854 schools of 99.03% of cities and towns of the country. The project works not only with the application of tests to discover talents for mathematics, but also provides the schools and their teachers with innovative teaching materials (problem sets). Moreover, a special program of supervised studies with scholarship of National Council for Research and Development (CNPq) of the Ministry of Science and Technology is offered to the 300 medalists of the Olimpiad. This supervised study is executed with the collaboration of research Universities around the country, and the entire material is specially designed and developed by researchers and specialists, with free access by school teachers as well as by the general public ([//www.obmep.org.br](http://www.obmep.org.br)). Many school teachers are taking benefits from this project. A recent paper (3) has studied statistically the impact of the participation of the schools in the OBMEP

on the average score points in mathematics of each participating school in Prova Brasil (INEP/MEC), also a large scale assessment of students' achievement, and it has shown the tendency to the improvement. Technical details can be found in (3).

Another important initiative for the professional development of mathematics teachers, illustrative of the increasing concern of the educators about the formation of future and in-service teachers of mathematics, can be found in the creation of many Post-Graduate Programs in both Mathematics and Mathematics Education. In particular, courses are designed specifically for Basic School Mathematics Teachers, at Master (and at least in one case Doctorate) level, in which attention is given to content knowledge together with classroom practices. Moreover, it can be noted that in at least one State of the Federation, a program of grants is available to teachers from the public school system who wish to study for a Master or Doctoral Degree. Government agencies that provide research funding for projects in different states of the country are also investing in special programs aimed at encouraging the participation of teachers from the public school in research projects aimed at improving all aspects of teaching and learning at the Basic Education level. These initiatives are reflections of the demands of a Basic Educational System that is struggling to overcome the many difficult issues rooted in socio-economic problems as well as in problems of inefficient content and methodological knowledge of teachers.

**Concluding remarks:** The above lines present an overview of the present state of the actions regarding the professional development of mathematics teachers in Brazil, although not comprehensive of all aspects. Nevertheless, they represent a dynamical move recently taken by diverse sectors of the Education System, from policy makers as Ministry of Education, researchers from Graduate Programs, to the educators and researchers involved in the ongoing projects. Moreover, it should be mentioned the initiative of a Group Study started in 2009, with the objective of discussing the mathematics content of curriculum of teacher preparation courses regarding the different levels of instructional practice. This group is intended to be a seminal initiative to grow as a steering discussion group to participate and contribute in ICMI activities. The Klein Project for 21st century launched by ICMI-EC is a fortunate opportunity for this group to include the questions raised by the Design Team in its agenda. All these initiatives represent hope for Basic Education in Brazil.

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## Annex 6. Systematizing knowledge about Mathematics Teacher Education – The IEA Teacher Education and Development Study in Mathematics (TEDS-M)

(With the collaboration of Gabriele Kaiser<sup>5</sup>)

As stressed in this document, the quality of mathematics teacher education is the key issue for ensuring quality mathematics education for all. This being more and more acknowledged, teacher education has become an area of considerable interest among researchers and practitioners as attested by the ICMI Study already mentioned (Ball & Even, 2009), the publication of an *International Handbook of Mathematics Teacher Education* in four volumes (Wood, 2008) and also policymakers in many countries, as attested for instance by the OECD study *Teachers Matter* (2005).

To date, nevertheless, despite the existence of many interesting studies analysing how the education of mathematics teachers is organized in different countries, investigating the nature of knowledge needed by the profession and how it develops, no empirical cross-national study based on representative samples has analysed how education systems prepare teachers of mathematics or identified the explicit and implicit expectations for what they should know as results of this preparation. Therefore, the IEA has set up an international study, which reflects the need to produce usable knowledge that will help inform policy to assist in the recruitment and preparation of a new generation of teachers as knowledge demands change and large numbers of teachers reach retirement age. This study, the Teacher Education and Development Study in Mathematics (TEDS-M) gathers data at the following three levels of teacher education systems across participating countries (see Tatto et al., 2008, p. 13f):

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5 Gabriele Kaiser is Professor for Mathematics Education at the University of Hamburg (Germany), and President of ICTMA (the International Community of Teachers of Mathematical Modelling and Applications), an Affiliated Study Group to ICMI.



1. “Outcomes: What is the level and depth of the mathematics and related teaching knowledge attained by prospective primary and lower secondary teachers? How does this knowledge vary across countries?”
2. Institutions and programs: What are the main characteristics of teacher education institutions and their programs? In what ways do these vary across countries? What are the learning opportunities available to prospective mathematics teachers (primary and lower secondary)? How are these structured (e.g., what is their level of internal/external coherence)? What content is taught in teacher education programs and is this instruction organized?
3. National policy: What is the national policy context for teacher education regarding, for example, recruitment, curriculum, quality assurance, and funding? How do these policies vary across countries?”

The overall goal of this study is to find better ways to help teachers learn what they need to know to teach mathematics. The different audiences are addressed as follows (Tatto et al., 2008, p. 15):

“In the case of educational policymakers, the aim is to suggest institutional and program arrangements that are effective in helping teachers become sufficiently knowledgeable. For teachers educators who design, implement, and evaluate teacher education programs, the primary aim is to give them a shared language and a shared database, and the shared benchmarks for examining their programs against what has proved possible and desirable to do in other settings. For mathematics educators, the purpose is to provide a better understanding of what qualified teachers of mathematics are able to learn about the content and the pedagogy of mathematics and the condition these teachers need to acquire this knowledge. For educators in general and for informed laypersons, the purpose is to provide a better understanding, backed by empirical research, about how and what teachers learn as they prepare to teach.”

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<http://www.iea.nl/teds-m.html>

## Annex 7. Research on mathematics teacher education in South Africa and Southern Africa

(With the collaboration of Jill Adler<sup>6</sup>)

Three major challenges impact on the quality of mathematics teacher education in South Africa in particular and Southern Africa in general:

- recruitment and retention of well qualified mathematics teachers, particularly at the secondary level;
- selection of contents for the preparation of teachers (their initial training) both in mathematics, in mathematics didactics and education; as well as for upgrading of teacher qualifications.
- the ongoing professional development and/or upgrading of in-service teachers to meet changing conditions (new technologies, new topics and orientations to mathematics).

Research related to the above issues is ongoing though relatively recent in the regional area, enabling deeper understanding of these challenges, and possibilities for addressing them. This reflects the current international situation where, even if exponentially increasing, research in mathematics teacher education is a relatively new field. Studies to date are largely small scale and 'insider' accounts, where research has been conducted by teacher educators in institutions where the research is being carried out. A review of such research has been reported in (Adler *et al*, 2005).

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6 Jill Adler is Professor of Mathematics Education at the University of the Witwatersrand, Johannesburg (South Africa) and at King's College, London (UK). She is also Vice-President of ICMI.

## **I. Recruitment and retention of mathematics teachers**

The report on mathematics teacher education in 12 African countries (Adler et al, 2007) provides details of issues of recruitment and retention across African countries. Movement of well qualified teachers across borders, though not well documented, is well known. There is open recruitment of newly trained graduates of South African universities to the USA and UK. And conditions in Zimbabwe have pushed large numbers of qualified Zimbabwe teachers to seek work in South Africa, and no doubt in other countries neighbouring Zimbabwe..These are further evidence of skills exodus from different countries and in different ways. These global forces are beyond the means of individual governments.

## **2. Content selections in initial teacher education**

It is widely accepted that the quality of an educational system depends on the quality (and one could add), stability of its teaching corps, and their teaching. Recent research in South Africa (Carnoy et al, 2008) confirms findings elsewhere (Hill et al, 2008), that student performance in mathematics is related to teachers' professional knowledge of mathematics, that is, their knowledge of mathematics per se, and the specialised knowledge of mathematics used in teaching. While only a pilot study, the Carnoy et al study provides evidence of positive correlations between the quality of teaching, teachers' mathematical knowledge for teaching, and student performance. This study adds to the understanding not only of the importance of initial teacher education, but to the calls for greater alignment between the content of mathematics teacher education and the mathematical work they do. Adler (2005) and Adler & Davis (2006) describe this specialised mathematics for teaching, and show that across institutions offering upgrading programmes for mathematics teachers in South Africa, specialised knowledge of mathematics is not assessed. They argue that this indicates both the important work required to develop improved understanding of such specialised competence and then how it is taught and assessed in both initial and upgrading programmes for mathematics teachers.

Innovation for initial training in a new undergraduate B Ed Degree in South Africa is underway, with attention to selections of mathematical content pertinent to teaching, as well as to how this is taught to prospective teachers. Such selections and integration is, however, not trivial. A recent study comparing the programmes and outcomes of two B Ed programmes (in a rural and urban University in South Africa) illuminates just how difficult this is, with effects on the quality of training offered (Parker, 2009). These programmes are, however, succeeding in attracting and training more mathematics teachers, including those at secondary level. In many institutions, the full responsibility of these programmes resides in Faculties of Education. A challenge to

Mathematics Departments, and the role of mathematicians in the preparation and support of teachers is currently being positively met by the various professional organisations in the country. Later this year, the Mathematical Society will meet with government and mathematics education to take this process forward. The potential of this meeting lies in the various participants being willing to recognise the diverse expertise required in and for mathematics teacher preparation and support.

### 3. Ongoing professional development and support of teachers in changing conditions

Adler & Reed (2002) report a study of a formalised professional development programme for mathematics, science and English language teachers, and illuminate just how critical is the distance between visions for educational reform and on the ground realities. Their findings support the discussion above, that content knowledge for teaching mathematics was not sufficiently addressed in the programme (despite intentions to do so). Moreover, their findings support the importance of understanding the evolution and time required for educational cultures and practices to change. They show further that in the poorer schools where the distance between on the ground realities of teaching and models for reform is very large, implementation of envisaged reform can function to worsen rather than improve teaching quality.

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## Annex 8. Promoting excellence in mathematics teaching –The National Centre for Excellence in the Teaching of Mathematics (NCETM)

[www.ncetm.org.uk](http://www.ncetm.org.uk)

(With the collaboration of Celia Hoyles<sup>7</sup>)

### Background

The NCETM was set up in England by the UK Government in 2006 as a national infrastructure for the support of the teaching and learning of mathematics by providing expert advice, resources and information. The National Centre oversees mathematics-specific continuing professional development (CPD) provision at a strategic level and coordinates its operation nationally. This is the first time such a national infrastructure has been put in place in England.

The National Centre aims to raise the professional status of all those engaged in the teaching of mathematics in order that the mathematical potential of learners will be fully realised. It takes as its starting point the premise that effective CPD has three interrelated strands:

- broadening and deepening mathematics content knowledge;
- developing mathematics-specific pedagogy, which includes appreciating how learners engage with mathematics and likely obstacles to progression; and,
- embedding effective mathematics pedagogy in practice.

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<sup>7</sup> Celia Hoyles is Professor at the School of Education, University of London, and Director of the NCETM. She is also a member of the ICMI Executive Committee.

Thus, **the key aims** of the NCETM are:

- to stimulate demand for mathematics-specific CPD contributing to strengthening the mathematical knowledge of teachers and improving school and college performance in mathematics;
- to lead and improve the coordination, accessibility and availability of mathematics-specific CPD;
- to enable all teachers of mathematics to identify and access high-quality CPD that will best meet their needs and aspirations.

## How the National centre works

The National Centre works with partners to promote CPD opportunities that are cumulative and sustained over the career of a teacher. Higher education institutions (HEIs), are important among these as they are already offering CPD opportunities for teachers. They can provide expertise in mathematics and mathematics education, as well as bringing new perspectives on the subject and effective pedagogies that will inform the work of the NCETM.

The National Centre has a virtual presence through its online web portal and an on-the-ground presence through a network of Regional Coordinators (RCs) that covers all of England. Each RC, along with part-time Advanced Skills teachers (that is, practising teachers who are designated as 'advanced' in their teaching and paid accordingly), works to create and support local networks of teachers within and cross phase. They encourage teachers of mathematics to engage in collaborative teacher enquiry, to identify opportunities for high-quality professional development and to share examples of excellence across the region. This includes supporting the spread of dynamic networks whereby teachers take the lead in developing their own communities – virtual and actual – thus both spreading ideas further and providing another and different type of CPD for teacher-leaders.

National and regional events play an important part in this work. For example, The Potential of ICT in Mathematics Teaching and Learning conference showcased the work of teachers using Information and Communication Technologies (ICT) in mathematics and explored its potential to raise standards, widen access to mathematical ideas, expand opportunities and narrow the gap in achievement.

## The NCETM portal

The NCETM's unique offer its web portal – [www.ncetm.org.uk](http://www.ncetm.org.uk) – a cutting-edge, online resource that allows the NCETM to reach those teachers that other more low-tech conventional means cannot reach. The portal signposts a wealth of excellent resources and is a dynamic means of sharing strategies for teaching mathematics through online networks and communities. Teachers of mathematics can also chart their individual CPD progress through the Personal Learning Space (PLS – see below for more on this). In brief, the portal includes:

- a Courses and Events directory
- a Resource section
- a Teacher Enquiry section where you can gain access to research developments and apply for NCETM funding for your own enquiry project
- the latest News on developments and issues in mathematics education
- Communities and Blogs, where teachers can share ideas about teaching
- the Mathemopedia, a wikipedia for mathematics education.

Other key portal features include the increasingly popular Secondary and Primary Magazines, with their accompanying Up2d8 Maths resources that explore a range of mathematical themes in a topical context. There is also an Early Years Focus, with news, tips and suggestions for work with this age group, and an FE magazine for post-16 and adult numeracy educators.

## The Personal Learning Space (PLS)

The PLS allows teachers to personalise their learning journey on the portal through linking to their favourite items, easily accessing their contributions, making notes and reflections on resources and materials and collecting evidence for use in their career progression.

## Self-Evaluation Tools (SETs)

The SETs are at the heart of the PLS. Teachers can assess their own mathematical knowledge and explore hundreds of targeted 'next steps' that provide appropriate ideas for taking their mathematics further, find examples of successful practice and discover how areas of mathematics can be linked. To date our feedback on the use of the SETs has been very positive. They are increasingly being used as part of regular performance review in schools and can be used to

help teachers establish which areas of the curriculum they might wish to work on. Many have reported that completing the self-evaluation as a whole department is very useful. (The NCETM has also developed number of departmental workshops aimed at whole teams, to provide structured professional development that can be delivered in-house. Each workshop module provides an overview and a number of resource sheets.) The SETs are also proving highly effective in HEIs, as a way for teacher trainees to self-evaluate and be guided to appropriate resources to improve their knowledge and practice.

## Registrations on the NCETM portal

There has been a consistent upward trend in registrations on the NCETM portal, which reflects the increasing richness of its provision as well as a growing awareness as to how it can be used effectively in schools, colleges and HEIs. The overall total of registered users now stands at 26 440: Primary and Early Years = 9 136; Secondary = 9 646; FE Colleges = 2 104; Sixth Forms = 1 147. Registered users are those who log in when they visit the site. In addition to these, huge numbers are visiting the portal without logging in (where they can still access a huge range of useful resources). The overall number of content hits on the portal reached nearly three and a half million by June 2009. Over a year, this has seen an increase of 147%.

## Reflections and challenges

There is still much to do and many challenges to face before CPD for all teachers of mathematics in the England is recognised as key and is universally demanded. The National Centre continues to seek to engage more teachers and senior leaders in all sectors and to find further ways to work with partners to help grow all aspects of provision. The Centre has to ensure that it has widespread impact on learners and – crucially – that this impact is recognized by politicians and policy makers.

Effective professional development offers teachers the chance to realise their full potential – enhancing their skills and aspirations, enabling them, in turn, to help all learners realise their own potential. This is so important for us all as individuals as well as for the UK as a whole. Recent reports have underlined the crucial role of mathematics in the UK's economic future and also in personal and professional confidence, as mathematics forms a key part of people's lives – at work and at home.

National Centre is helping to bring about a shift in UK culture where no longer will it be socially acceptable to say, 'I can't do mathematics – and I am even proud of it.' The NCETM is passionate about ensuring that there are enough well-qualified and confident teachers of mathematics in all England's schools and colleges and providing them with the high-quality CPD support they need.



## Annex 9. Experiencing Mathematics! – An international travelling exhibition

(With the collaboration of Michel Darche and Mireille Chaleyat-Maurel<sup>8</sup>)

Following the 2000 Mathematics Year, UNESCO, in line with its interest in education and in promoting international cooperation, brought together mathematicians from France, Japan and the Philippines, already involved in the popularisation of mathematics, to build the international exhibition *Experiencing Mathematics*. Developed with the support of several different institutions, including the International Mathematics Union (IMU) and its commission for education (ICMI), the exhibition was first presented at the 10th International Congress in Mathematics Education, in July 2004 in Copenhagen, Denmark.

The exhibition then began to take a tour around the world, as mathematicians, mathematics teachers and educators from different countries asked for it. Particular attention was paid to presenting it in developing countries, by privileging the circulation of the exhibition by continental regions, such as Southern Africa and Latin America, which is more economic for the countries petitioning and enriching for the exhibition.

### Aims of the exhibition

The exhibition is mainly addressed to those between the ages 10-18, but also to their teachers and parents. It has been conceived with three main objectives:

- to raise public awareness and interest in mathematics, to demonstrate not only that mathematics is indispensable and everywhere but that it is interesting, challenging and fun as well;
- to demonstrate that mathematics is within everyone's reach, that, conversely to what is generally assumed, a good grasp of basic mathematical concepts can be achieved by the majority, and that important mathematical ideas can be made widely accessible.

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8 Michel Darche is Directeur Honoraire of the Centre Sciences, CCSTi Région Centre-Orléans, and Mireille Chaleyat-Maurel is a Professor at Paris V University, France, and co-chair with Minela Alarcon from UNESCO of the Exhibition *Experiencing Mathematics!*

- to conceive all the experiences so that teachers would be able to use them in their classrooms. A key component for their training!

It was designed and produced by the “Centre Sciences” in Orleans, France.

### Assessment over five years

During the last five years (2005-2009), the exhibition has been presented in 90 cities of 32 different countries in East and West Africa, China and East Asia, Latin America and Europe. It has welcomed nearly **1,200,000 visitors**, of whom about 75% were young visitors, and more than **15,000 teachers and tutors**. Everywhere, the presentation was prepared by a **training session for teachers** and animators during three days on average. These presentations have been well covered by the media (newspapers, radio and TV, buzz on many websites), and provided an opportunity for organizing specific activities linked to mathematics (such as workshops, lectures and competitions).

### Some illustrative examples:

- in Namibia (2006), 50,000 pupils and teachers in 12 cities in 3 months;
- in Madrid, for the ICM2006, nearly 40,000 visitors in 3 months (with queues outside in the summer holidays) and students from more than 100 academic establishments;
- in 2007, in Bangkok, Laos (4 cities), Viet-Nam (2 cities) and Cambodia (4 cities), and Singapore, 120,000 young visitors with 4 specific training sessions for teachers around the usefulness of Mathematics in Cambodia sponsored by IMU, ICMI, the CIMPA and UNESCO;
- in India (2008), nearly 100,000 visitors in 4 towns (Delhi, Calcutta, Bangalore, Mumbai) with 2 specific training sessions for teachers organised by the local UNESCO office;
- in Pakistan (2008), 50,000 young visitors in 3 cities with the National Science Foundation;
- in Latin America (2008-2010): 8 countries, a tour beginning and ending in Chile and including Brazil (10 cities in 7 months), and a second time in Chile, in January 2010, for a two weeks training session in mathematics for 2,000 teachers with the University of Chile.

All these presentations were carried out with the logistic and effective support of the Ministries of Education and Research, Mathematical Societies, Embassies, Science Foundations of each country and Science Museums of each city.

To provide information about the exhibition and associated events, its circulation, and its partners, a website was created: [www.MathEx.org](http://www.MathEx.org)

### The virtual exhibition

To complement the exhibition, a virtual exhibition in four languages (English, Spanish, French, Portuguese) was conceived two years ago with the financial support of the UNESCO office in Southern Africa (Namibia and Angola). It includes more than 30 virtual and interactive experiences, and 300 activities are proposed as mathematical experiences. The virtual exhibition also presents examples of educational use of the exhibition themes and associated printed documents for teachers, in order to reinforce its educational impact, especially in developing countries.

These virtual tools are conceived so that teachers can use them in their classrooms without the aid of computers, but with only usual material such as paper, string, cardboard and glue, and experience mathematics with their students. It also aims to favour the development of material showing how the themes of the exhibition can find resonance in different cultures. It has been diffused on 300 CD-ROMs, and is freely accessible on the Internet: [www.ExperiencingMaths.org](http://www.ExperiencingMaths.org),

It was presented and used in two specific training sessions for teachers in Angola (INIDE) in 2008 and 2009 (more than 100 CD-ROMs were disseminated in Angola, and three to five given in all countries where the actual exhibition was presented). A presentation can be downloaded from the Volume 4.3 of *Matemática*, a digital Spanish journal on mathematical popularization: [www.matematicalia.net/index.php?option=com\\_wrapper&Itemid=410](http://www.matematicalia.net/index.php?option=com_wrapper&Itemid=410)

### New project, new website:

#### What careers need maths? What jobs use maths?

The public in general, pupils and even mathematics teachers have little information about outlets offered to mathematics graduates of any levels. They generally believe that research and teaching are the only careers accessible to such graduates. It is thus necessary to inform parents and pupils at an early age in schools.

To support this goal, in some places, mathematical events linked to the exhibition have been organized, like shows of mathematical movies, conferences of professional mathematicians and even theatre plays related to mathematics.

We now aim to develop, in connection with the exhibition, a new project offering new tools to young people, their teachers and their parents raising awareness about the diversity of possible jobs obtainable through science education:

- an original interactive website with games, reports, interviews (in written or video format),
- posters or flyers at the exhibition, with some portraits of young professionals who use a great amount of mathematics in their jobs, but also showing a variety of professions accessible with different levels of mathematics qualifications, paying the necessary attention to different kinds of balance between genders and regions, as well as between centres and peripheries.

We plan to take advantage of having the exhibition in a specific region to interview well-known mathematicians of the area, whose reports on the specific situation and mathematical needs in their country could progressively enrich the set of accessible resources.

## Annex 10. Challenging Mathematics – Mathematics Houses in Iran

In this annex, we briefly present the goals and main activities developed by mathematics houses, a structure created during the last decade in Iran. These perfectly illustrate what can be done in the framework of non-formal education, when the different communities interested in mathematics and mathematics education develop a productive collaboration.

As recalled in (Barbeau & Taylor, 2009, p.88), the origin of mathematics houses in Iran results from the creation of a high commission headed by the President of Iran for the observance of the 2000 World Mathematical Year set up in 1997. This commission indeed took as a goal the creation of mathematics houses. The first one opened in Isfahan in 1999. To date, mathematics houses exist in Isfahan, Neishabour, Tabariz, Yazd, Kerman, Khomein, Kashmar, Sabzevar, Babul, Zenjan, Gazvin, Gonbad and Najafabad, and a specific commission has been established for organizing cooperation between them.

Mathematics houses have six main goals:

1. popularizing mathematics;
2. studying the history of mathematics;
3. studying the applications of mathematics, statistics and computer sciences;
4. developing information technology;
5. expanding mathematical sciences among young students;
6. promoting team work among young students and teachers.

These goals are achieved through:

- procuring facilities for non-conventional education;
- introducing new educational techniques;
- establishing scientific data banks;
- encouraging joint and collaborative research;
- modeling and applying mathematical sciences;
- welcoming relevant novel ideas.

A diversity of activities serving the general public, students of all levels and their families, teachers and even university professors, graduate students, researchers and artists, are organized by the mathematics houses. We list these in the following paragraph, relying on the presentation made in (Barbeau & Taylor, 2009, pp. 88-92) and on a text written by Ali Rejali for the ICMI Bulletin on the occasion of the tenth anniversary of the Isfahan Mathematics House (IMH) (Rejali, 2009). This very active mathematics house is an especially insightful example. More information can be found in its website: [www.mathhouse.org](http://www.mathhouse.org)

### Activities organized by IMH include:

1. Lectures (both on popular and special topics in mathematics and mathematics education). For instance, every year, there are five or six public expository lectures and many special talks for special groups of students, teachers and members of the house.
2. Mathematics and information technologies exhibitions. Special “days” and “weeks” are regularly organized around such exhibitions. More generally, the mathematics houses provide computer facilities where participants can use and develop software, access the Internet and benefit from electronic resources for learning mathematics.
3. Activities for highschool students. These are quite diverse and include research groups which present the results of their investigations in annual festivals or in publications, mathematics team competitions for instance in the frame of the International Tournament of Towns, the Isfahan school net which establishes electronic communication for schools and provide information technology for education and research, robotics workshops, camps and problem-solving workshops.
4. Activities for university students: statistics day, research groups involved in collaborative research through electronic communication with Iranian researchers abroad, entrepreneurship for giving students the opportunity of designing web pages and software, introductory workshops to the use of mathematics and statistics software.
5. Activities for teachers: research groups in various educational fields, information technology workshops to train teachers in the use of modern educational devices and familiarize them with information technology, workshops on goals, standards and concepts of mathematics education for elementary teachers, on new secondary courses and information technology for secondary teachers.

At IMH, moreover, a group of researchers is developing specific activities for teaching mathematics and computer sciences to blind students. Furthermore, IMH and some other mathematics houses

maintain specialized libraries providing access to resources of interest regarding mathematics education available in the country.

Mathematics houses cooperate between themselves, but they also collaborate with various Iranian institutions such as the Adib Astronomy Centre, the Iranian Mathematical Society, the Iranian Statistical Society, the Isfahan Mathematics Teachers' Society, the Iranian Association for Mathematics Teachers' Societies, the Scientific Society for Development of Modern Iran; the Isfahan Society of Moje Nour for the blinds, and the Science and Art Foundation. New forms of cooperation are emerging with some other foreign institutes such as Fontys and the Freudenthal Institute in the Netherlands, or in France the association Animath coordinating the diversity of existing non-formal educational activities in mathematics and the IREM network (*Institut de Recherche sur l'Enseignement des Mathématiques* – Institute of Research on Mathematics Education)<sup>9</sup>.

In no more than one decade, mathematics houses in Iran have already achieved a lot, and they are receiving increased international recognition.

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9 For more information about Animath and non-formal educational activities in mathematics in France, see (Zehren & Bonneval, 2009). For information about the IREMs, see Annex I I.

## Annex II. Collaboration between mathematicians, teachers and researchers in mathematics education – the case of the IREM network

The IREM network (Institute of Research on Mathematics Education – *Institut de Recherche sur l'Enseignement des Mathématiques*) is a network that has progressively developed since the late sixties in France. The first three IREM were created in 1969 in order to meet the needs for teacher education and development resulting from the New Math reform. Others soon followed and IREMs quickly covered the entire country. There are now 28, one per educational region (the so-called academies), plus a few IREM created abroad recently.

IREM constitute a unique university structure within which researchers in mathematics and mathematics education, teacher educators, elementary and secondary school teachers work together. Each maintains close relations with the mathematics department of their university and nobody works full time in an IREM.

Their mission is threefold:

- contributing to the initial preparation of teachers and their continuous professional development;
- conducting research and innovation in mathematics education;
- producing resources for mathematics teaching and teacher training.

The assembly of IREM Directors (ADIREM) and fifteen inter-IREM Commissions focusing on different educational levels or themes federate the IREM network. This network has a Scientific Committee involving researchers working in IREM and eminent scientists from outside the network.

The IREM network publishes a journal: *Repères IREM*, addressed to mathematics teachers, and several IREM also publish journals of national or even international audience such as the journals *Grand N* (focused on elementary education) and *Petit x* (focused on middle school education and also open to science education) published by the IREM of Grenoble, or *Annales de sciences cognitives et didactique* published by the IREM of Strasbourg. Together with APMEP (*Association*



*des Professeurs de Mathématiques de l'Enseignement Public*), the network has also in charge the francophone database Publimath.

In France, research in mathematics education initially developed mainly within the IREM and this has undoubtedly contributed much to the importance given in this research right from the beginning to the mathematical dimension of didactic research, on the one hand, and to classroom research and didactical engineering, on the other.

Since their inception, IREM have supported the principle of teacher education being both close to the terrain and nurtured by research, be it didactic, epistemological or historical, the improvement of connections between disciplines and the collaboration between different communities, and they have helped develop productive synergies between these. They have played an undeniable role in the evolution of elementary and secondary education through the participation of their members to various national commissions in charge of curriculum design or reflecting on mathematics education, by conducting pre-experiments of curricular changes when possible, by being a force for proposals and issuing critical advices, and by developing teacher training programs to accompany curricular reforms.

This innovative structure is a fragile one, however, primarily because of its singularity (there is no equivalent structure for any other discipline). Moreover, even if it is perceived positively by the various institutions having responsibility in mathematics education in France, it is now affected by a drastic reduction in its resources, especially those allowing the participation of secondary school teachers to the IREM activities under good conditions, and by the insufficient attention paid to in-service teacher education.

For more information, consult the IREM portal: <http://www.univ-irem.fr/spip.php>

## Annex 12. The emergence of a teacher community – the case of Sesamath

(With the collaboration of Gérard Kuntz, Benjamin Clerc and Sébastien Hache<sup>10</sup>)

We stressed in the document the need to rely on types of functioning other than those usually used to promote and support the necessary changes in teaching practices. We also emphasized the potential role of information and communication technologies in these evolutions. From this point of view, the case of the association Sesamath (<http://www.sesamath.net/>), created in 2001 by a handful of French mathematics teachers convinced of the importance that new technologies would have in the learning and teaching mathematics, is an interesting one to analyze. Eight years after its creation, Sesamath indeed occupies a central place today in France and in the Francophone world in connection with the creation of online free and open resources. A few data suffice to provide evidence of that fact: 1,300,000 hits per month on its site, 15,000 mathematics teachers having registered for its Newsletter, 6,000 teachers registered on the private site Sesaprof created in 2008, 500,000 students enrolled in the Mathenpoche-network, and all this with a body of 71 members. We present in this annex some important features of the project Sesamath that, in our opinion, help to understand this achievement. We also emphasize the observed evolution in its relationship with existing institutions and the research world. We rely particularly on the joint contribution (Kuntz, Clerc & Hache, 2009) presented at the EMF 2009 Conference.

Since its inception, the project Sesamath has been based on some strong convictions:

- that information technologies are changing radically the capacity of teachers to create resources, to share, test and improve them through collaborative work inside communities of practice;
- that the direct connection with teachers in the field is a priority, even before the mediation of teacher and researcher organizations, and that the regulation permitted by critical users can ultimately ensure the quality of resources produced;

<sup>10</sup> Gérard Kuntz is a member of the Scientific Committee of the IREMs, Benjamin Clerc and Sébastien Hache are founding members of the association Sesamath.

- that only a site quickly offering many resources and covering a large part of the curricula is viable in this context and likely to federate a critical mass of teachers.

The community of practice that has federated around this project was small, strongly connected, and offered diverse and complementary competencies. It first created a communication tool: a website dedicated to contacting the French mathematics teachers, inviting them to share resources that each had created alone. From the outset, many proposals were received, generating visibility and exchanges, and stimulating the reflection of the initial group. Specific projects emerged, especially the database of exercises Mathenpoche (Mep), which soon fully covered the middle school syllabus.

The article (Kuntz *et al.*, 2009) attempts to identify the reasons for such a success and mentions the following: a project conducted by practicing teachers, a strong expertise in computers, a long-term and ambitious vision, rigor of organization, a permanent dialogue from a distance, and finally a very active core group whose size remains limited.

The Sesamath association originally developed its project in direct contact with targeted users and under their control, but outside traditional mediators in France such as APMEP (*Association des Professeurs de Mathématiques de l'Enseignement Public*), IREM (*Institut de Recherche sur l'Enseignement des Mathématiques*) and the community of researchers in mathematics education. The choices made, and in particular that of quickly covering the syllabus for the four years of middle school, have led to resources usable but easily subject to criticism. And criticisms were sharp, reinforced by the very success achieved by Mep, by mistrust with respect to a community of teachers developing their project independently and progressing so rapidly, and also by the reservations of many teachers, researchers and inspectors regarding this type of technological resource for education.

In fact, by 2004, a rapprochement with the IREM occurred, leading to the creation of a commission inter-IREM/Mathenpoche, later renamed "Online Resources". Fairly soon, too, links were established with researchers in mathematics education who began to analyze the exercises proposed in Mep on specific areas, suggested improvements and also tried to identify the uses made of Mep by teachers and the effects of these uses on students' learning.<sup>11</sup> Doing so, they discovered a listening quality, responsiveness and adaptability that they generally did not expect. This is consistent with the vision of Sesamath founders, who see Sesamath resources not as objects that are released only after being patiently developed, tested and improved, but as objects more quickly shared, not pretending to be optimal, but thought in such a way to

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11 The group ECUM (Emergence of Communities of Mep Users), for instance, has studied, from September 2006, the experimentation of Mep in the academy of Rennes. Its observations and conclusions are accessible on the website EducMath ([http://educmath.inrp.fr/Educmath/lectures/dossier\\_mutualisation/ecum](http://educmath.inrp.fr/Educmath/lectures/dossier_mutualisation/ecum)).

evolve and be adapted continuously through collaborative work. Many questions result from this experience for which answers require the development of appropriate research.

Today Sesamath, all of whose products are free, is open to international cooperation: translation into Spanish of Tracenpoche in Peru, which already leads to the local creation of many geometry exercises for use in the Hispanic world that might feed a local version of Mep and will in turn enrich Mep-France; and a UNESCO / OIF / AUF project of transfer of Sesamath competencies to Senegal and Mali which is now in the phase of finalization. Worldwide cooperation is thus emerging, for which the potential results are still unpredictable.

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## **Annex 13. Fostering interactions and collaborations “Teacher Education Around the World: Bridging Policy and Practice”, a component of the IAS/Park City Mathematics Institute, Institute for Advanced Study, Einstein Drive, Princeton, New Jersey**

(With the collaboration of Herb Clements and Gail Burill<sup>12</sup>)

In the document, we have stressed the necessity of fostering collaborations between mathematicians, mathematics educators, teachers and policy-makers, and also organizing international collaborations, sensitive to the diversity of contexts and cultures. The Park City Mathematics Institute and its International Seminar provide an original and interesting example of action copying with this ambition. We briefly present it in this annex, focusing more especially on the International Seminar.

The IAS/Park City Mathematics Institute (PCMI) is home to the seminar “Teacher Education Around the World: Bridging Policy and Practice” held annually in Park City, Utah, as part of the annual PCMI Summer Session, since 2001. The seminar itself is funded by the Wolfensohn Family Foundation and the Bristol-Myers Squibb Foundation.

As mentioned above, the PCMI International Seminar on Mathematics Education is a somewhat unusual contribution to the international mathematics education scene. The one week seminar has been held every year since 2001, except during the years (2004 and 2008) of the International Congress of Mathematical Education (at which past participants meet to share recent initiatives from their countries with each other and with the international mathematics education community). The function of the Seminar is to design and implement a series of

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12 Herb Clements is Professor at Ohio State University, USA, Former director of IAS/Park City Mathematics Institute, and Vice-Chair of the Committee for Developing Countries of the International Mathematical Union. Gail Burrill is Professor at Michigan State University, USA, and co-chair of the PCMI International Seminar.

reflections on common problems, suggestions for policy and practice, or innovative offerings that are made available to the international community. The 32 countries represented over the last eight years come from all continents of the world and represent a mixture of cultures and traditions as well as a balance of developing and 'developed' nations. The program is one part of a larger set of summer PCMI programs designed to serve mathematics researchers, mathematics graduate students, undergraduate faculty, undergraduates and secondary teachers, which meet at the same site for a three week summer session, sharing a variety of activities across programs.

The goals of the international seminar are to:

- promote open discussion of issues affecting the mathematics education policies and practices of each nation;
- identify common issues faced across national contexts;
- develop a sense of international shared purpose in the pursuit of quality mathematics education;
- search for common solutions to related problems.

Each year the participants come as teams consisting of one mathematics educator/policy-maker and one practicing secondary mathematics teacher from eight countries, representing each geographic region of the world. In order to ensure some continuity in the group from one year to the next, foreign teams are often involved two consecutive years. The program engages participants in a discussion of issues from the dual lenses of educational policy and day-to-day practice, in particular examining policies and practices related to the preparation and development of mathematics teachers. Formal presentations are restricted; instead a mathematical content theme (for instance Functions in 2009) is used to focus conversations on general cross cutting topics such as:

- the relationship of national standards and national curriculum to teaching practice in classrooms in each country;
- the system of teacher education in each country and how it relates to teaching practice;
- the way each country and culture deals with the challenges of excellence and accessibility in mathematics education;
- the role of mathematics education as a profession and of mathematics education research in each country.

Policy briefs addressing common issues that arise in the discussion related to the theme and the context of its enactment in different cultures are prepared and published on the web

([mathforum.org/pcmi/int.html](http://mathforum.org/pcmi/int.html)), and a continuation of the dialogue is then organized through the PCMI/MathForum list serve. The long-range goal is to share the panel's information, outcomes and conclusions with the public at large, through articles in the mass-circulation press and professional journals.

There is no doubt that this symposium should be of compelling interest to educational policy-makers and researchers, thanks to:

- the model of international dialogue and communication which it represents;
- the juxtaposition of policy and actual classroom outcomes, explored in a spirit of trust and shared purpose;
- the distillation of its proceedings and outcomes;
- the subsequent dialogue it stimulates.

Of no less importance, it can also contribute to the recognition by policymakers and the public of the importance of:

- professional norms and professional working conditions for teachers;
- and quality career-long teacher professional development

as necessary conditions for improving mathematics teaching worldwide.

Another interesting feature of this action is the productive interaction between mathematicians, mathematics educators, teachers, policy-makers, that the inscription of the International Seminar in the PCMI structure allows. It can certainly be a source of inspiration for other projects. Note that plans have been made for developing regional actions obeying a similar structure in connection with PCMI, first in Uganda then in Cambodia, in collaboration with ICMI and CIMPA, but that they could not be implemented up to now due to insufficient funding.

**Participating countries (from 2001 to 2009):** Australia, Brazil, Cambodia, Cameroon, Chile, Columbia, Denmark, Ecuador, Egypt, France, Germany, India, Iran, Israel, Japan, Kenya, Mexico, Namibia, Netherlands, New Zealand, Northern Ireland, Pakistan, Peru, Poland, Romania, Russia, Singapore, Sweden, Turkey, Uganda, USA, Vietnam.

**More information is accessible at:** [mathforum.org/pcmi/int.html](http://mathforum.org/pcmi/int.html)

## Annex 14. Rebuilding a mathematical community in Cambodia

(With the collaboration of Michel Jambu<sup>13</sup>)

This is a presentation of the Cambodian project introduced in 2005 by the International Centre for Pure and Applied Mathematics (ICPAM) in order to create a community of mathematicians holding doctorates. It is based on a report written by Michel Jambu. The main goal of the project is to support mathematics education at Bachelor and Master degree level by modernizing the Cambodian curriculum. It will be considered successful when enough young researchers, having followed this curriculum and having obtained a doctorate in a foreign country, return to Cambodia and take charge of mathematics education.

This ICPAM project has received considerable support, namely from the Agency of Francophone Universities (AUF), the International Mathematical Union (IMU), and from UNESCO under the International Basic Sciences Programme (IBSP). Several educational institutions (such as the universities of Paris VI, Marseille and Nice, INSA Rouen Engineering School, the Institute of Mathematics of the Vietnamese Academy of Science and Technology (VAST) and the Faculty of Sciences of Sfax) have also contributed. Gradually, new partnerships, not only French but also American, Japanese, Swedish and German, have been formed. It must be noted that Japanese support was provided by the Toyota Foundation and American support came from the US National Committee for Mathematicians. Following Professor H. Clemens' visit to Cambodia in 2007, the International Mathematical Union (IMU) decided to open a Volunteer Lecturer Programme (VLP) in which professors teach advanced mathematical courses in the Cambodian project as well as in a similar programme in Uganda. This programme will be extended to other developing countries.

However, as outlined in Michel Jambu's report, nothing could have been achieved without the invaluable and constant help from Dr Chan Roath. Such a project could not take place without a local and efficient partnership.

From 2005 to 2007, the project was located at the Royal Academy of Sciences of Cambodia (RAC), before it was transferred to the Royal University of Phnom Penh (RUPP). The Master degree truly began in November 2007, with the registration of 25 students who are now

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13 Michel Jambu is Professor at the University of Nice and former Director of CIMPA.



finishing the second year of their Master degree, followed by the registration of 11 students in 2008.

The Rector of RUPP decided to open the course to a new group of students only every two years, owing to the low number of students (11 in 2009) and the high cost of this programme and difficulties in obtaining funding for the foreign professors teaching this Master degree (18 in 2009). When conditions improve, the course could perhaps be opened to a new group of students every year.

The scientific programme gives a basic and general background while trying to maintain a balance between the necessary fundamentals and the more applied courses in statistics, numerical analysis and mathematics for engineers. Nine courses were held in 2007-8 and 15 in 2008-9, with eight in the Master 1 and seven in the Master 2. The 2009-10 programme in Master 2 comprised 11 courses, each lasting three weeks, totalling 495 hours.

The report highlights the importance of student access to a library with sufficient reference work. Various actions have been taken by ICPAM in this connection, such as the purchase of books at a special rate following agreements between publishers (Hermann and SpringerVerlag) and ICPAM. Paul Vaderlind and Rikart Bogvad (ISP, Sweden) also provided electronic access to around 900 mathematic books. This is just the beginning.

The report mentions the expectation that around ten PhD graduates should return within four to five years. These mathematicians could then take charge of teaching the Bachelor and Master degrees. The foreign partnership would then change and aim to maintain the contact of these young researchers with the international scientific community. It is also highlighted that these young researchers would have to obtain positions at RUPP, with wages and working conditions that would allow them to continue their research. In particular, it stresses the fact that they would need post-doctorate scholarships in foreign countries and that should they have to teach too many hours in order to earn a decent wage, as it is currently the case, they would not be able to undertake research after a number of years and all this work would be partially wasted.

Several years' preparation is necessary for the integration of these young PhD graduates in the Cambodian university system. They will have the responsibility of training young students, organizing academic programmes in accordance with the socio-economic needs of the country and training managers in the private sector, as mathematics are needed in the development of most sectors.

Moreover, mathematics education reform in Cambodia is planned under a World Bank project for the modernization of the Cambodian education system. The report points out the fact that the education given in a Master degree is the link that could contribute to it, but that this is not enough as long as young PhD graduates do not return to Cambodia in sufficient

numbers and under adequate working conditions. He insists on the fact that the modernization of mathematics education must be part of this project, in universities also, by including Master and Bachelor degrees.

This action clearly shows the complexity of such a rebuilding as well as what can be achieved by solidarity within the international community when it can rely upon organizations such as ICPAM. It also shows that nothing can be done without strong collaboration with the local forces and without supporting them in their efforts. The creation of a mathematics community in Cambodia, as in any country, is a necessary condition for the education of qualified teachers, and this education is, itself, as we have highlighted in this report, a necessary condition for improving basic education.

**For more information, see the ICPAM website:**

[www.cimpa-icpam.org](http://www.cimpa-icpam.org)

# Annex 15. List of participants at the Experts' Meeting

30 March – 1 April 2009

## EXPERTS

NAME	TITLE	AFFILIATION/ADDRESS
<b>M<sup>r</sup> Michèle Artigue</b>	President of the International Centre for Pure and Applied Mathematics (ICPAM)	Université Paris Diderot – Paris 7 175-179 rue Chevaleret Plateau E, 6 <sup>e</sup> étage 75013 PARIS Université Denis Diderot 2, place Jussieu Case 7018, 75251 PARIS Cedex 05
<b>Mr Christian Buty</b>	Lecturer in Education Sciences Head of the "Learning, Didactics, Interactions, Knowledge" for Science team	Institut national des recherches pédagogiques (INRP), France
<b>Mr Didier Dacunha-Castelle</b>	Professor Emeritus Researcher	Faculté des Sciences d'Orsay Université Paris-Sud 11 bureau : 102 bât : 425 F-91405 ORSAY Cedex
<b>Mr Pierre Lena</b>	Delegate to education and training	Académie des sciences 3, quai de Conti F-75270 Paris Cedex 06 France
<b>Mr Peter Okebukola</b>	Professor of sciences Pro-Chancellor of Teaching and Chairman of the Board Former Vice-Chancellor Lagos State University, Nigeria Former Secretary of the National Universities Commission of Nigeria	Crawford University Faith City, Igbesa, Ogun State Nigeria
<b>Mr Juan Ignacio Pozo</b>	Director of the Psychology Department Representative of the UNESCO Chair in Science Education in Latin America and the Caribbean	Catedrático de Psicología Básica Facultad de Psicología Avenida Ivan Paulov, 6 Universidad Autónoma de Madrid 28049 Madrid España
<b>Mr Charles Ryan</b>	Lecturer, Science and Teacher Education University of Winchester	The Gables, Kiln Road, Redlynch, Salisbury, Wiltshire SP5 2HT, UK
<b>Mr Baruch Schwarz</b>	Lecturer, Institute of Education Hebrew University of Jerusalem, Israel Associate Professor at Intermedia, University of Oslo, Norway	Hebrew University Harav Zinger, 7, Givat Shmuel Israel Intermedia, University of Oslo Intermedia Vilars Lomell Postboks 1161, Blindern 0318 Oslo, Norway
<b>M<sup>r</sup> Merle Tan</b>	Director	National Institute for Science and Mathematics Education of the University of the Philippines
<b>Mr Jesus Vazquez-Abad</b>	Professor and Research Chair in Science and Technology Education	190 Willowdale # 603 Montréal, Québec CANADA H3T 1G2
<b>Mr Mario Wschebor</b>	Mathematician, former President of the Executive Committee of the International Centre for Pure and Applied Mathematics (ICPAM)	Centro de Matemática. Facultad de Ciencias, Universidad de la República Calle Iguá 4225. 11200 Montevideo, Uruguay

## UNESCO

NAME	TITLE	AFFILIATION/ADDRESS
<b>Mr Qian Tang</b>	Assistant Director-General for Education	Education Sector
<b>Ms Linda King</b>	Acting Director Division for the Promotion of Basic Education	Education Sector
<b>Mr Maciej Nalecz</b>	Director of Division SC/BES	Natural Sciences Sector
<b>Ms Beatriz Macedo</b>	Programme Specialist ED/BAS/STV Coordinator of the meeting	Education Sector
<b>Ms Minella Alarcon</b>	Programme Specialist SC/BES	Natural Sciences Sector
<b>Mr Ary Mergulhao</b>	Programme Specialist	UNESCO Brasilia

## RAPPORTEURS

**Ms Isabelle Merkovic**  
UNESCO ED/BAS/STV

**Ms Magalie Lebreton**  
UNESCO SC/BES

Today we live in a world that is profoundly shaped by science and technology. Scientific and technological developments have never been as rapid nor had such a significant impact on our societies, whatever their state of development. The major challenges that the world has to face today – concerning health, the environment, energy and development – are both scientific and human challenges. To meet these challenges, the world needs scientists who are able to imagine the kind of future that we might only just perceive, and to make that future happen; but the world must not reserve the understanding of these challenges and the debate on proposed developments to an elite.

Today, nobody can doubt that positive, sustainable and equitable development may be obtained without the support and contribution of the great majority. Therefore, nobody should doubt that the challenge of shared intelligence, of quality education for all, and especially quality science education for all, including mathematics and technology education, is the only way forward. Without such education, it is meaningless to speak of civic debate and participation.

This publication thus identifies the challenges to be met to ensure quality mathematics education during basic education and, on the basis of case studies, suggests ways of improving it. It will be useful both to policy-makers seeking to incorporate quality science and mathematics education into their education systems and to the various actors who wish to participate in the process of change.



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