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Global Content Framework of Reference for Mathematics: Global Consultation

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Summary

Purpose of Global Consultation

This paper presents the results of the global consultation on the Global Content Framework of Reference for Mathematics. Aiming to reach a global consensus on the Global Content Framework, UIS and IBE-UNESCO set up an online consultation, which focused on the first two levels of the Global Content Framework: domains and sub-domains. Participants were asked to test the Global Content Framework by using it to map their country's National Assessment Framework (NAF) at the domain and sub-domain levels. An online platform was thus developed¹.

Participants were asked to map local assessment frameworks onto corresponding DOMAINS and Sub-DOMAINS in the Global Content Framework. In total, 6 domains and 16 sub-domains were included. Participants completed the survey indicating using yes/no responses whether or not Global Content Framework categories were present in local documents. Following this activity, respondents were asked to reflect on their experience while they completed a 9-item survey.

Outcomes

Twenty-six survey responses were gathered and uploaded for analysis. Three additional blank forms contained no usable data (Kenya, U.S.A., Finland). From among completed forms, 14 were prepared by national bodies, 3 by international agencies and 5 were either independent consultants' reports or responses from university groups. One completed survey lacked any indication of group affiliation.

- The majority of the participants found the description provided regarding the development of the Global Content Framework to be satisfactory, the mapping process to be simple, and the troubleshooting section to be helpful.
- When asked how they intended to use the Framework, it was interesting that only one participant selected "Mapping National Assessment Frameworks (NAFs)" as a stand-alone option, although this was the activity everyone was asked to complete in the Global Content Framework. Most (12 or 46%) indicated interest in using the Global Content Framework to map either the first two (4 or 33%) or all three (8 or 67%) options: curriculum, NAFs, and curriculum-NAF alignment.
- When asked to indicate clarity of the steps in the mapping process, responses varied. In the cases where respondents found the steps to be confusing, this was due to differences between local documentation and the categories of the Global Content Framework and because the mapping process was unaccounted for theoretical assumptions in the Global Content Framework.
- Frequencies of response rate to the question "Which categories of the NAF that you used for mapping, mapped well on the Global Content Framework, and which ones did not?" indicate that most respondents ascribed support for categories which mapped well. Categories which did not map well mostly included sub-domains.

¹ The online platform is accessible here: <http://ibe-unesco.org/a-global-framework-for-mathematics/>

Recommendations

- **The Global Content Framework should include more detailed information than presented in the survey so that it can flexibly address different users/uses**
The construction of a more detailed and flexible Framework was suggested, that would provide sufficient context for users to determine fit of local mathematics information, as well as clarification of the role of the Global Content Framework, when dealing with local documents, with instructive examples.
- **Fundamental theoretical assumptions underlying the Global Content Framework model need to be addressed**
There is no question the final Framework, indeed any framework, would benefit from additional theoretical support. However, this benefit is of marginal use only if we can address practical issues related to the manifestation of new theoretical ideas. The Global Content Framework is based on a theory of mathematics cognition, one that helps to organize, compare and make sense of curricular and assessment materials in an arbitrary way. Its value rests in its usefulness to end users; a point that should be amplified. The report recommends the development of instructive examples and description of the purpose of the Global Content Framework more clearly and fully.
- **Format of the Framework/Survey should be improved**
The report suggest the development of an iterative questionnaire design (i.e. a design that slowly changes with time) to gather information about users, their intentions when engaging the Global Content Framework, their experiences while using the Framework, and the utility of the Global Content Framework with respect to local needs. The questionnaire design strategy should also include items that get at users' responses to thorny issues.
- **Cultural and/or language differences present barriers when using the Global Content Framework**
The report recommends the addition of items to an iterative questionnaire design strategy that get at language/culture/social information.

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Simplest model view showing only DOMAINS. Each DOMAIN is accompanied by a brief description. Use the 1-level model to map the most general categories included in national assessment frameworks and curriculum documents.

#	DOMAIN	Description:
1	Mathematics Proficiency	understanding and using various approaches to problem solving, reasoning, communicating understanding and results, negotiating solutions to tasks
2	Number Knowledge	understanding and using pre-number ideas (e.g., counting), symbols, and different number systems (e.g., whole numbers, fractions)
3	Measurement	understanding and using non-standard units (e.g., pencil lengths, teacup amounts) and standard units (e.g., inches, grams, litres) to measure various quantities
4	Statistics and Probability	understanding and using good data management procedures (e.g., organizing, representing, interpreting) to conduct investigations; using chance and probability experiments (e.g., coin tosses) to explore mathematics of probability
5	Geometry	understanding and using properties of 2-D shapes (e.g., lines, triangles) and 3-D objects (e.g., cubes, spheres), completing transformations (e.g., rotations, reflections), and working in the Cartesian plane (e.g., plotting points)
6	Algebra	understanding and using non-numerical patterns (e.g., patterns observed in the environment), numerical patterns (e.g., sequences), functions (linear and non-linear), and properties of variation (i.e., ratio, proportion, percent)

Simplest model view showing DOMAINS and Sub-DOMAINS. Each Sub-DOMAIN is accompanied by a brief description. Use the 2-level model to map categories included in national assessment frameworks and curriculum documents.

#	DOMAIN	#	Sub-DOMAIN	Description
1	Mathematics Proficiency	1.1	Problem Solving	demonstrate understanding, ability to plan, do and check work during solution of a mathematical problem
		1.2	Reasoning	recognize various problem elements associated with a task, recognize and use concepts and procedures to assist with solution attempts, justify approaches, concepts and procedures used
		1.3	Communicating	using appropriate mathematical vocabulary, connecting ideas to everyday life, interpreting mathematical statements
2	Number Knowledge	2.1	Pre-Number Ideas	learning to count (e.g., 1 to 10), basic number sense (e.g., cardinality), number rhymes, songs, using concrete materials
		2.2	Numbers and Number Systems	natural numbers, fractions, decimals, integers, sets and set theory, exponents, alternative number systems (e.g., Roman numerals)
3	Measurement	3.1	Non-Standard Units	measuring quantities in the environment using non-standard measures (e.g., pencil lengths, book-weights, teacup volume, long times)
		3.2	Standard Units	measuring quantities in the environment using standard measures (e.g., inches, meters, Litres, grams)

4 Statistics and Probability

4.1 Data Management

creating surveys and questionnaires, administering them, collecting data, summarizing and representing data, interpreting results

4.2 Chance and Probability Experiments

recognizing and using principles of chance to make predictions, principles of probability and simple probability experiments (e.g., coin tosses)

5 Geometry

5.1 Geometric Shapes and Objects

constructing geometric shapes and objects (e.g., lines and angles, plane figures), using properties of geometric shapes and objects to solve problems

5.2 Position and Direction

translating, rotating, reflecting and dilatating various geometric shapes and objects

5.3 Properties of Space

locating geometric shapes and objects in the Cartesian plane

6 Algebra

6.1 Non-Numerical Patterns

recognize and use patterns in the environment and culture

6.2 Numerical Patterns

recognize and use numerical patterns in sequences and algebraic expressions to solve problems

6.3 Functions

recognize and use linear and non-linear functions to solve problems

6.3 Vectors

recognize and use properties of vectors and arithmetic operations involving vectors to solve problems

6.4 Variation

recognize and use ratio, percentage and proportion to solve mathematical problems

Model view showing three levels, DOMAINS, Sub-DOMAINS and Constructs. Note that some Sub-DOMAINS are also defined as Constructs (e.g., Mathematics Proficiency). Each Construct is accompanied by a brief description. Use the 3-level model to map categories included in national assessment frameworks and curriculum documents.

#	DOMAIN	#	Sub-DOMAIN	#	Construct	Description
1	Mathematics Proficiency	1.1	Problem Solving	1.1.0	Problem Solving	demonstrate understanding, ability to plan, do and check work during solution of a mathematical problem
		1.2	Reasoning	1.2.0	Reasoning	recognize various problem elements associated with a task, recognize and use concepts and procedures to assist with solution attempts, justify approaches, concepts and procedures used
		1.3	Communicating	1.3.0	Communicating	using appropriate mathematical vocabulary, connecting ideas to everyday life, interpreting mathematical statements
2	Number Knowledge	2.1	Pre-Number Ideas	2.1.1	Number sense	counting concrete objects, number words, number games, rhymes
				2.1.2	Operations with objects	grouping and taking away concrete objects from a collection of objects
		2.2	Numbers and Number Systems	2.2.1	Natural numbers	counting and operations with positive integers including zero (i.e., 0, 1, 2, 3, ...)
				2.2.2	Fractions	counting and operations with rational numbers expressed as a/b where a is the numerator and b is the denominator; b does not equal 0
				2.2.3	Decimals	counting and operations with real numbers expressed in base ten notation
				2.2.4	Integers	counting and operations with negative and positive real numbers (i.e., ... -2, -1, 0, 1, 2, ...)
				2.2.5	Sets	counting and operations with finite or infinite collections of objects or numbers where order is of no importance
				2.2.6	Exponents	operations with numbers where a quantity (the base) is raised to the power of another quantity (the exponent)
				2.2.7	Alternate number systems	number systems other than base ten

3

Measurement

3.1 Non-Standard Units

3.1.1 Shapes and objects

measuring quantities in the world using locally derived units (e.g., book-lengths, spoon volumes, stone-weights)

3.1.2 Daily Living

measuring quantities in your daily life using locally derived units (e.g., estimating time duration)

3.2 Standard Units

3.2.1 Shapes and Objects

measuring quantities in the world using established measurement units (e.g., inches, cm, km, L, gallons, lbs, kg)

3.2.2 Daily Living

measuring quantities in daily life using established units (e.g., time in months, days, hours, mins; currency; temperature)

4

Statistics and Probability

4.1 Data Management

4.1.0 Data Management

creating surveys and questionnaires, administering them, collecting data, summarizing and representing data, interpreting results

4.2 Chance and Probability Experiments

4.2.0 Chance and Probability Experiments

recognizing and using principles of chance to make predictions, principles of probability and simple probability experiments (e.g., coin tosses)

5

Geometry

5.1 Geometric Shapes and Objects

5.1.1 Constructions

constructing lines, angles, plane figures, 3-D objects; investigating symmetry and congruence

5.1.2 Properties

recognize and use properties of lines and angles, plane figures, 3-D objects, symmetry and congruence and similarity

5.2 Position and Direction

5.2.0 Position and Direction

translating, rotating, reflecting and dilatating various geometric shapes and objects

5.3 Properties of Space

5.3.0 Properties of Space

locating geometric shapes and objects in the Cartesian plane

6

Algebra

6.1	Non-Numerical Patterns	6.1.1	Relations	investigating patterns in the environment (e.g., colours, shapes, sounds) and patterns in cultural activities (e.g., local handicrafts, dance, music, mosaics)
6.2	Numerical Patterns	6.2.1	Relations	investigate patterns in number sequences, investigate properties of algebraic expressions
6.3	Functions	6.3.1	Linear functions	recognize and use appropriate algebraic notion, properties of linear functions, linear equations, simultaneous equations
6.3	Functions (cont)	6.3.2	Non-linear functions	recognize and use appropriate algebraic notion, properties of non-linear functions and solve non-linear simultaneous systems
6.4	Vectors	6.4.0	Vectors	recognize and use properties of vectors and arithmetic operations involving vectors to solve problems
6.5	Variation	6.5.0	Variation	recognize and use ratio, percentage and proportion to solve mathematical problems

Model view showing three levels, DOMAINS, Sub-DOMAINS and Constructs. Note that some Sub-DOMAINS are also defined as Constructs (e.g., Mathematics Proficiency). Each Construct is accompanied by a brief description. Use the 3-level model to map categories included in national assessment frameworks and

#	DOMAIN	#	Sub-DOMAIN	#	Construct	#	Sub-Construct	Description
1	Mathematics Proficiency	1.1	Problem Solving	1.1.0	Problem Solving	1.1.0.1	understand	describe problem situations, differentiate between implicit and explicit problem information, derive additional (unstated) information relevant to the problem (e.g., use graphs), formulate one or more questions to describe the problem
						1.1.0.2	plan	model problem situations from everyday life, translate language-based descriptions into mathematical statements, select from among various concrete, visual and symbolic means, recognize various representational forms (e.g, graphs, pictograms), select appropriate approaches given problem conditions,
						1.1.0.3	do	anticipate solution trajectory of a problem, estimate order of magnitude of the final result, apply various problem solving strategies (e.g., applying previous knowledge, trial-and-error, 4-step strategy--understand, plan, do, check)
						1.1.0.4	check	evaluate strategies and verify results, evaluate selected models to determine appropriateness of solution approach, describe solution approach with precise mathematical language, compare computed and expected results, assess efficiency of solution trajectory, justify approach taken
				1.2.0.1	problem elements	develop personal approaches to mental and written problem solving, connect various representational forms, derive personal heuristics, rules, laws, and procedures to approach new problems (e.g., use counter-examples, classify problem elements)		
					Reasoning		Reasoning	

Mathematics Proficiency

1.2	Reasoning (cont)	1.2.0	Reasoning (cont)	1.2.0.2	concepts and procedures	construct tables, figures and diagrams; recognize and account for initial conditions; estimate first then compute results; formulate questions and pose possible responses to address initial assumptions; evaluate reasonableness of conjectures by continuously monitoring solution approach	
			1.2.0.3	justification	recognize problem situations in everyday life; explain reasoning orally and in writing using appropriate mathematical language; explain results using accepted definitions, axioms, properties and theorems; identify errors and explain possible causes; propose corrections for errors; recognize situations where technological solution approaches could be applied		
	1.3	Communicating	1.3.0	Communicating	1.3.0.1	using mathematical vocabulary	explain procedures and results using appropriate language, representations and symbols; use explanations of processes, statements and solutions to deepen understanding; communicate thinking orally, visually and in writing; distinguish between everyday meanings and mathematical meanings; use different sources of information to communicate findings to others; evaluate arguments made by others to deepen understanding
				1.3.0.2	connecting to everyday life	summarize information and processes and results to make them more comprehensible to others; discuss mathematical statements, approaches, and procedures with others	
				1.3.0.3	interpreting mathematical statements	explain solutions to problems using various representations (e.g., drawings, tables, graphs); select mathematical language to suit context, message and audience; associate images, objects and concepts with mathematical terms, symbols and representational forms in context	

Number Knowledge

2.1

Pre-Number Ideas

2.1.1 Number sense

2.1.1.1 counting concrete objects

sorting or matching objects; determining cardinality of a set of objects; reading, writing, and singing numbers (to 10); counting forward/backward; skip-counting with words; recognizing what zero (the empty set) represents

2.1.2 Operations with objects

2.1.2.1 grouping and taking away concrete objects

using principle of conservation of numbers when counting objects; grouping collections of like objects; adding and taking away objects; practising with simple addition/subtraction sentences; solve simple missing-value problems

Numbers and Number Systems

Natural numbers

2.2.1.1 counting with symbols

read and write any number; order number words and number symbols; describe relative number location (i.e., equal to, less than, greater than); sort numbers in increasing/decreasing order; count in sequence backward/forward; skip-count forward/backward (e.g., by 1's, 2's, 5's, 10's)

2.2.1.2 counting along the number line

locate any number on the number line; sort numbers by magnitude; skip-count forward/backward; describe relative location of a number (using $<$, $>$, $=$)

2.2.1.3 properties

use cardinality/conservation of number to count everyday objects; use zero to signify absence of magnitude; identify numbers using different representations; identify and use even/odd, prime/composite, perfect numbers; estimate numbers to the nearest 10, 100, 1000, 10 000 using different representations (e.g., number line, diagram); approximate numbers (i.e., estimate to a given digit, round up/down) based on known properties (e.g., even/odd numbers); use place value to identify magnitudes of digits; compose/decompose numbers using distributive/associative properties; identify equivalent number expressions (e.g., $52 = 40 + 12$ or $25 + 27$); identify properties of prime/composite numbers; compute multiples of natural numbers (e.g., 5's: 1×5 , 2×5 , ...); determine factors of any given number (e.g., $24 = 6 \times 4 = 2 \times 3 \times 2 \times 2$); find prime factors of a given number (e.g., $9 = 3 \times 3$); find lowest common multiple (LCM) and greatest common factor (GCF)

Number Knowledge (cont)

Numbers and Number Systems (cont)

2.2.1

Natural numbers (cont)

2.2.1.4

adding and subtracting

calculate sums/differences; estimate sums/differences; compute sums/differences with regrouping (e.g., $35 + 47 = (30 + 5) + (40 + 7)$); describe various strategies for computing sums/differences (e.g., using number facts, commutative property, associative property, decomposition, estimation, inverse relation between addition/subtraction); create and solve your own addition/subtraction question; solve missing value problems (including combined addition/subtraction problems); evaluate reasonableness of results by checking results

2.2.1.5

multiplying and dividing

write multiplication/division facts (to 12×12); compare product/quotient statements using $<$, $>$, $=$; calculate/estimate products/quotients (with/without borrowing, carrying); calculate products/quotients with order of operations; describe various strategies (e.g., facts, concrete/pictorial representations, grouping, repeated addition/subtraction, inverse relationship between operations, decomposition, estimation, distributive property); recognize and use special properties of 0 and 1 when computing products/quotients; explain the relationship between multiplication and division (i.e., $D = d \times c + r$); decompose numbers into prime factors; evaluate reasonableness of results by checking

2.2.1.6

combined arithmetic operations

calculate/estimate combined operations questions with order of operations; describe various strategies (e.g., concrete/pictorial representations, grouping, repeated addition/subtraction, facts, inverse relationships between operations, decomposition, estimation, distributive property); determine precision of results (e.g., approximate, truncate, round up/down); evaluate reasonableness of results by checking; explaining problem solving approach to others

Number Knowledge (cont)

Numbers and Number Systems (cont)

2.2.2

Fractions

2.2.2.1 properties

recognize fraction as equal parts of the whole using concrete objects/diagrams; identify different representations for fraction using concrete objects; read and write symbolic and diagrammatic representations for fractions; represent selected fractions symbolically (e.g., $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{2}{4}$, $\frac{3}{4}$, $\frac{1}{8}$); order fractions with the same denominators by magnitude (using $<$, $>$, $=$) in symbols or along the number line; define fraction as a rational number $\frac{a}{b}$ where $b \neq 0$; use place value to interpret multi-digit fractions; approximate fractional values (e.g., $\frac{14}{26} \approx \frac{1}{2}$), identify subsets of rational numbers

2.2.2.2 equivalencies

identify equivalent expressions (e.g., $52 = \frac{104}{2}$); compose/decompose fractions using distributive/associative properties; use GCF to find fraction equivalents (i.e., lowest form); describe equivalency properties between proper, improper fractions and mixed numbers; order fractions expressed in lowest form by magnitude; identify fraction equivalents between proper and improper fractions and mixed numbers; convert between fraction equivalents; interpret measurement quantities (e.g., lengths, heights, mass, price) expressed as fractions

2.2.2.3 addition and subtraction

compute/estimate sums/differences between fractions and fraction equivalents with or without same denominators (e.g., $1 + \frac{1}{2}$, $\frac{1}{4} + \frac{6}{5}$); describe various addition/subtraction strategies (e.g., concrete/pictorial representations, grouping, LCM/GCF, multiplication/division facts, decomposition, estimation, distributive property)

2.2.2.4 multiplication and division

express multiplication/division fraction facts concretely/pictorially and symbolically; compute/estimate products/quotients of combinations of fractions and natural numbers (e.g., $\frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$; $\frac{1}{4} \times 3 = \frac{3}{4}$; $\frac{1}{2} \div 2 = \frac{1}{4}$; $\frac{1}{2} \times 2 \frac{1}{4} = \frac{9}{8}$); express computation of products and quotients in lowest terms; describe various multiplication/division strategies (e.g., concrete/pictorial representations, grouping, LCM/GCF, facts, decomposition, estimation, distributive property)

Number Knowledge (cont)

Numbers and Number Systems (cont)

Fractions (cont)

2.2.2.5

combined arithmetic operations

compute/estimate combined operations involving fractions, fraction equivalents and natural numbers while observing order of operations and simplifying results; select strategies to solve problems and defend them; determine appropriate precision of results (e.g., approximate, truncate, round up/down); evaluate reasonableness of results

Decimals

2.2.3.1

properties

represent decimals in various ways (e.g., concrete/pictorial or symbolic representations); read and write decimals (up to 4 decimal places); apply place value to decimals to describe magnitudes of all digits; compose/decompose decimals using associative/distributive properties (e.g., $0.425 = 0.4 + 0.02 + 0.0005$; 17×0.025); approximate/truncate/round (up/down) decimals; sort decimals by magnitude using knowledge of place value; compare magnitudes of decimals, fractions and natural numbers; order decimals, natural numbers and fractions by magnitude along the number line

2.2.3.2

equivalencies

write various decimal equivalents for fractions and vice versa (e.g., $\frac{1}{4} = 0.25$); order decimal equivalents by magnitude (up to four decimal places); convert proper and improper fractions and mixed numbers to decimals (and vice versa); express fractions with powers of ten in the denominator as decimals and vice versa (e.g., $\frac{4}{10^3} = \frac{4}{1000} = 0.004$); express percentages as decimals, fractions and fraction equivalents and vice versa (e.g., $\frac{3}{2} = 1.5 = 150\%$); express everyday measures (e.g., length, mass, volume) as decimals (e.g., 2.4 cm, 0.45 m^2)

2.2.3.3

addition and subtraction

compute/estimate sums/differences between decimals using knowledge of place value and, where appropriate, order of operations; describe various strategies to find sums and differences between decimals (e.g., addition/subtraction facts, commutative/associative properties, decomposition, estimation, inverse relationships between operations)

2.2

2.2.3

Number Knowledge (cont)

Numbers and Number Systems (cont)

2.2.4

Decimals (cont)

2.2.3.4

multiplication and division

compute/estimate products/quotients of combinations of decimals natural numbers and integers (e.g., $1.24 \times (-2) = -2.48$; $0.13 \times 0.13 = 0.0169$; $0.45/-3 = -0.15$); describe various strategies to find products and quotients of combinations of decimals, natural numbers and integers (e.g., facts, commutative/associative property, decomposition, inverse relationship between operations)

2.2.3.5

combined arithmetic operations

compute/estimate results for tasks involving decimals and decimal equivalents, combined arithmetic operations and order of operations; describe various strategies when solving combined operations problems (e.g., concrete/pictorial representations, grouping, repeated addition/subtraction, number facts, inverse relationship between operations, decomposition, estimation, distributive property); round results to given number of decimal places; express results as fractions and percentages and vice versa; select a solution approach and defend it; determine suitable level of precision to present results (approximating/truncating/rounding where appropriate); evaluate reasonableness of results by checking; explain problem solving approach to others

Integers

2.2.4.1

properties

represent integers in various ways (e.g., concrete/pictorial/symbolic); count backwards/forwards across zero to investigate positive and negative numbers; read and write any integer; compute selected integers by subtracting natural numbers; locate/order integers on the number line by magnitude (i.e., using $=$, \neq , $<$, $>$, \leq , \geq); represent integers as fractions, decimals and percentages; express integers as rational numbers (i.e., $a = b/c$); approximate integers by rounding, truncating, estimating; apply absolute value to interpret distance between integer numbers

Number Knowledge (cont)

Numbers and Number Systems (cont)

2.2.5

Sets

Integers (cont)

2.2.4.2

combined arithmetic operations

compute/estimate results for combined operations involving any combination of integer and decimal/fraction equivalents (e.g., $\frac{1}{2}$ [1.5(-3 + 4)]); describe strategies to solve combined operations problems (e.g., concrete/pictorial representations, grouping, repeated addition/subtraction, number facts, inverse relationships between operations, decomposition, estimation, distributive property); solve combined operations problems involving integers and fractions/fraction equivalents while observe rules governing order of operations; select and defend your approach; evaluate the reasonableness of the results.

2.2.5.1

properties

recognize elements and non-elements of sets; identify characteristics of set elements given concrete/pictorial representations of sets; compare two sets (i.e., equal to, as many as, etc); recognize 1-to-1 correspondence between elements of related sets; create various sets from everyday objects and number symbols; construct enumerative sets from their descriptive forms and vice versa; distinguish between equal and equivalent sets; recognize the universal set and complementary set; describe properties of selected number systems using sets (e.g., set of natural numbers, whole numbers, rational numbers); recognize properties of the set of natural numbers; determine subsets of a set (up to 5 elements); use rule for computing number of subsets ($\# \text{ subsets} = 2^n$); determine LCM and GCF in selected sets

2.2.5.2

union and intersection

describe union and intersection of concrete objects in language and pictorially; graphically represent union and intersection of sets (i.e., using Venn diagrams); list elements that are members of unions and intersections (of up to 3 sets); find union, intersection and differences for up to three sets; compute Cartesian product of two sets (up to 3 elements each)

Number Knowledge (cont)

Numbers and Number Systems (cont)

2.2.6 Exponents

2.2.6.1 powers

relate multiplication of natural numbers by powers of ten with exponentiation; identify components of a power (i.e., base, exponent); represent selected powers (e.g., $2^3 = 2 \times 2 \times 2$); read and write numbers in exponential form using positive integral exponents (e.g., 5^4 , 9^2); recognize that any power with a zero exponent is equal to 1; compute exact magnitudes of powers with natural number/fractional base and positive integral exponent (e.g., 6^2 , $(\frac{1}{2})^2$); express powers using scientific notation (e.g., $482 = 4.82 \times 10^2$); recognize place value of powers to solve contextual problems; decompose exponents into prime factors (e.g., $2^4 = 2^{(2 \times 2)} = 2^2 \times 2^2$); estimate orders of magnitude of real numbers using powers (e.g., 123 is approximately $10^2 + 4^2 + 2^2$); decompose powers into prime factors (e.g., $147 = 3 \times 49 = 3 \times 7^2 = 3 \times 7 \times 7$); order powers, fractions, decimals, percentages, square roots, scientific notation along a number line; solve problems using product rule [i.e., $a^n \times a^m = a^{(n + m)}$]; solve problems using quotient rule [i.e., $a^n \div a^m = a^{(n - m)}$]

2.2.6.2 roots

estimate various square roots (e.g., $26^{1/2}$ is approximately 5); represent selected square roots of natural numbers concretely/pictorially/symbolically; compute exact values of roots with selected positive fractional exponents (e.g., exponents $\frac{1}{2}$, $\frac{1}{3}$); use the square root key on the calculator to compute exact values; simplify square roots by factoring (e.g., $\sqrt{12} = \sqrt{4 \times 3} = 2\sqrt{3}$); simplify roots by rationalizing the denominator (e.g., $3/\sqrt{2} = 3\sqrt{2}/2$); compute exact results for combined fractions, surds and multiples of pi problems.

Alternate number systems

2.2.7.1

alternative number bases

read and write vigesimal numbers (up to 5-digits); convert vigesimal numbers to base ten (and vice versa); convert sexagesimal numbers to base ten; convert base two/base five numbers to base ten; read and write local number symbols to 100 (base ten; e.g., Ghanaian numbering)

Number Knowledge (cont)

Numbers and Number Systems (cont)

2.2.7

Alternate number systems (cont)

2.2.7.2 Roman
numeration

read and write Roman numerals (up to M); compare Hindu-Arabic numbers and Roman numerals; recognize historical importance of zero in base ten vs. Roman number systems

2.2.7.3 Mayan
numeration

read and write Mayan numbers (up to 400); recognize position and functionality of zero in base ten vs. Mayan numbering; identify preceding/proceeding numbers given any Mayan number; order Mayan numbers by 20's and 100's; recognize significance of 1, 4, 5, 7, 13, and 20 in the Mayan cosmos; read and write Mayan dates in the Mayan calendar; investigate Mayan words for various fractions

2.2.7.4 arithmetic
operations

add/subtract vigesimal numbers (up to 4-digits); add/subtract Mayan numbers with carrying (up to 160 000); multiply Mayan numbers where one number is between 2 and 19 and the other number is 2-digits

Measurement

Non- Standard Units

3.1

3.1.1

Shapes and objects

3.1.1.1 length

measure lengths of everyday objects using personally devised units (e.g. pencil lengths, foot lengths); construct rulers from local materials and use them to measure everyday objects; describe distance between objects, people and places using locally derived units; locate objects in space relative to the four cardinal points

3.1.1.2 perimeter and
area

measure perimeter/area of drawn squares/rectangles using locally devised units; estimate perimeter/area of drawn squares/rectangles using locally devised units

3.1.1.3 volume/capacity

describe volume/capacity using everyday language; measure/estimate volume/capacity of concrete objects using locally devised units

3.1.1.4 angles

compare various angles observed in the environment using locally derived units

3.1.1.5 mass

describe weight of objects using everyday language (e.g., heavy vs. light); measure/estimate mass of concrete objects using locally devised units; describe loads on horses using culturally relevant units

Measurement (cont)

Non-Standard Units (cont)

3.1.2 Daily Living

3.1.2.1 time

describe duration of events using everyday language (e.g., long vs. short)

Standard Units

Shapes and Objects

3.2.1.1 length

measure/estimate length/height (imperial, metric) of everyday objects (e.g., km, m, cm, mm; mile, yard, foot, inch, rod); order various length units by magnitude using $<$, $>$ and $=$; appropriately convert between units (e.g., km to m; miles to yards) given specific problem situations; convert between imperial and metric units (e.g., km to miles; feet to dm) in specific problem situations; round results to given number of significant digits; compare standard and non-standard length units (to understand importance of standardizing measures); construct drawings to scale using various units

3.2.1.2 perimeter and area

describe characteristics of perimeter in regular and irregular figures found in the environment; measure perimeter (imperial, metric) of regular and irregular polygons (e.g., counting grid squares, direct measurement); measure lengths of line segments in plane figures (e.g., diagonals, circumference, radius, diameter, arcs, segments) resulting from isometries or similarity transformations; describe characteristics of area in squares and rectangles (e.g., # grid squares, # tiles in software application); compute/estimate area (imperial, metric) of various regular and irregular polygons (feet^2 , in^2 ; m^2 , cm^2 , mm^2); select appropriate area units to solve problems in context; convert between imperial and metric area measures; construct rectangles given perimeter and/or area information; classify characteristics of rectangles with the same perimeter and/or area to solve problems in context; compute area of plane figures resulting from an isometry or similarity transformation; use relation between area, length and width in rectangles to solve problems in context

Measurement (cont)

Standard Units (cont)

3.2.1

Shapes and Objects (cont)

3.2.1.3 surface area

surface area

compute surface area of various cubes, parallelepipeds, right prisms, right cylinders and right pyramids (manually or using software); compute surface area of selected complex solids that can be subdivided into elemental geometric forms (e.g., faces that are triangles, base a square); calculate surface area of a sphere, total and lateral area of a right cone (and any complex solid that can be subdivided into spheres and cones); justify the choice of area units and statements about units; discuss sources of possible measurement error

3.2.1.4 mass

mass

sort two or more everyday objects by weight (e.g., heavier/lighter); measure/estimate mass (imperial, metric) of everyday objects (kg, g; lbs, ounces); convert between equivalent masses (e.g., kg to g; pounds to ounces); convert between equivalent imperial and metric masses (e.g., kg to lbs); select appropriate mass units given a specific problem context; compute sum/difference in mass of up to three objects; round results to given number of significant digits; describe possible sources of measurement error

3

Measurement (cont)

3.2

Standard Units (cont)

Shapes and Objects (cont)

3.2.1.5 volume/capacity

describe characteristics of volume/capacity of everyday objects (e.g., # cubes to fill a cuboid); measure/estimate capacity/volume (imperial, metric) of everyday objects (L, mL, cm^3 , mm^3 ; gallons, quarts, ft^3 , in^3); select appropriate units to measure volume/capacity given a specific problem context; convert among equivalent imperial and metric volume/capacity units (e.g, mL to L; quarts to gallons); convert between equivalent imperial and metric volume/capacity units (e.g, quarts to L; mL to in^3); sort various volume/capacity units by magnitude (using $<$, $>$ and $=$); compute capacity/volume (imperial, metric) of rectangular prisms, cubes and parallelepipeds by relating to simpler forms (e.g., # cuboids making up a more complex figure); compare volume/capacity of everyday objects in selected problem situations; describe possible sources of measurement error; compute sum/difference in volume/capacity of up to three objects; round results to given number of significant digits

3.2.1.6 angles

estimate angle measures by comparing against 45 and 90 degree magnitudes; measure angles to the nearest degree using a protractor; classify angles by magnitude (i.e., acute, right, obtuse); describe characteristics of various types of angles (e.g., complementary, supplementary, adjacent, vertically opposing, alternate, interior/exterior, corresponding); measure various types of angles; measure interior angles in triangles; measure central angles and angles in arcs; justify statements about angles

Measurement (cont)

Standard Units (cont)

3.2.2

Daily Living

3.2.2.1 time

measure/estimate time intervals associated with everyday events (e.g., weeks, days, hours); sequence time duration of various events (e.g., compare amount of time to complete daily activities); use time equivalencies to solve problems in context (e.g., # min in an h); tell time on analogue and digital clock faces; tell and write time in various ways (e.g., Roman numerals, 12- and 24-hour clocks); select appropriate time units given a problem situation; estimate duration to complete events and verify findings by measuring; distinguish between duration and position in time (include concept of negative time)

3.2.2.2 schedules and calendars

organize your personal activities into a schedule; interpret time-lines and calendars for traditional and school-related events; sequence events over time using everyday language (e.g., days of the week, months of the year, significant dates); use time equivalencies to solve problems in context (e.g., days in a week, seconds in an hour); describe time patterns (e.g., # days in each month, # days in a week); identify names of days, weeks and months in the Mayan calendar; describe important dates for cultural events using the Mayan calendar; use kumatzin to compute days (Mayan calendar); compute the long count (K'im, Winai, Tun, K'atun, Baktun) for different dates in the Gregorian calendar; compare Mayan and Gregorian calendars for corresponding dates

Measurement (cont)

Standard Units (cont)

Daily Living (cont)

3.2.2.3 money

recognize different denominations in local currency; combine currency denominations to make specified amounts (e.g., \$1.45 = \$1.00 + \$0.25 + \$0.1 + \$0.1); use play money to make purchases and sell things in a play store; read and write various denominations and combinations of denominations (e.g., ten cents = \$0.1; a dollar forty five cents = \$1.00 + \$0.25 + two \$0.1); convert between different denominational units (e.g., \$1.00 = 4 x \$0.25); evaluate personal property in terms of local currency; solve simple profit/loss problems and interpret results with respect to the context; compute wages/salaries; explain various bank transactions and services; explain types of insurance and compute insurance premiums; explain income tax and national goods and services tax; use international currency equivalencies to solve problems in context (i.e., national currency, US dollar, EURO, other currencies); round computations to a given number of significant digits

3.2.2.4 temperature

estimate ambient and outside temperature using everyday language; measure/estimate temperature in decimal notation (Fahrenheit, Celsius); use temperature equivalencies to convert between Fahrenheit and Celsius scales; use integer quantities to interpret the temperature scale (above/below zero)

Statistics and Probability

4.1

Data Management

4.1.0

Data Management

4.1.0.1

data organization

create survey questions to address questions about you and your environment (e.g., school gender statistics, ages, heights); collect/record data from survey questions using variety of approaches (e.g., interviews, focus groups); select appropriate sampling method (e.g., simple random, systematic); recognize data types (i.e., discrete vs. continuous); categorize data with respect to two or more attributes (e.g., colour and size of a collection of marbles); organize data from different sources into appropriate intervals (e.g., plant types and growth measurement waves); define sample, population and variable; select representative samples from various populations; describe limitations of sampling; recognize sources of bias

4.1.0.2

data representation

display data using various means (e.g., line plots, bar graphs, pictograms, tally charts, tables, stem-and-leaf plots, histograms, pie charts, bar charts, block graphs, scatter plots); represent data in tables, graphs and histograms as frequencies/percentages; compute central tendency (mean, median, mode) from ungrouped data; display central tendency from ungrouped data using such things as stem-and-leaf plots and line graphs; identify and display min/max data values; construct tables and line graphs for longitudinal data; compare single variable distributions from up to two data sources; display bivariate data using scatterplots; draw a line of best fit to represent central tendency in bivariate data

4

Statistics and Probability (cont)

Data Management (cont)

Data Management (cont)

4.1.0.3 data interpretation

interpret survey/questionnaire results from data tables, pictograms, bar graphs, double bar graphs, line graphs, broken-line graphs, and circular graphs; verify assumptions about survey information (e.g., examining collection methods, sampling methods, representational methods); interpret data expressed as frequencies/percentages; interpret central tendency from ungrouped data; interpret data range (min/max); select appropriate ways to interpret and present findings; explain connections between central tendency and data range (e.g., importance of outliers to interpretation); evaluate interpreted results (i.e., by comparing representations, justifying choices of representations, and recognizing that graphs can be manipulated to support particular arguments); compare related data sets (i.e., in tables and/or graphs); interpret and evaluate graphs appearing in the media; interpret diagrams for grouped discrete and continuous data; interpret single variable distributions from various sources; interpret mathematical relationships between two variables; infer attributes of the population using statistical methods; interpret tables and graphs representing longitudinal data; predict data trends (interpolation, extrapolation)

Statistics and Probability (cont)

Chance and Probability Experiments

Chance and Probability Experiments

4.2.0.1 chance

recognize that there is variability in possible outcomes of an experiment; make predictions about everyday events (e.g., coin toss, dice rolls); recognize situations where outcomes are equally likely; recognize importance of independent events in a series of experiments; enumerate all possible outcomes of an experiment and express them as frequencies; compute probability as the chance of a particular event (expressed as fractions, decimals or percentages); define probability limits (i.e., 0 to 1); differentiate between probabilities using everyday language (e.g., certainty that event will/will not occur, possible); create theoretical sample spaces for single and combined events (e.g., equally likely and mutually exclusive outcomes); interpret differences/similarities between two sample spaces using dot and stem-and-leaf diagrams; describe the likelihood of combinations of possible events; compute the likelihood of an event using combinatorics

Statistics and Probability (cont)

4.2

Chance and Probability Experiments (cont)

4.2.0

Chance and Probability Experiments (cont)

4.2.0.2 probability

conduct simple experiments (e.g., coins, dice); collect and categorize results of random experiments (e.g., dice or coins) using tables, bar charts, stem-and-leaf plots, and line graphs; define sample space in a simple random experiment; describe possible outcomes of a simple random experiment; distinguish between predictions and outcomes; distinguish between theoretically predicted and experimentally derived outcomes in a simple random experiment; predict frequencies of outcomes based on theoretical probabilities; compare probabilities of different events without computation; compute probability of a particular outcome in a simple random experiment; use computed probabilities to inform decision-making; predict outcomes of future experiments based on a probability model (recognizing that with increasing sample size empirical samples tend toward theoretical distributions); conduct repeated random experiments (e.g., repeated coin tosses) and propose trends; compute independent and dependent probabilities of combined events using tree diagrams and understanding assumptions; calculate and interpret conditional probability (i.e., represent expected frequency with two-way tables, tree diagrams and Venn diagrams)

Geometry

Geometric Shapes and Objects

Constructions

5.1.1.1 lines and angles

draw lines and rays (e.g., straight lines, curved lines, continuous and dotted-lines; construct horizontal/vertical and parallel/perpendicular lines using pencil and ruler; draw plans in perspective (architectural) to investigate parallel and perpendicular lines in the environment; bisect line segments using a ruler and pencil; classify angles in everyday objects by magnitude (e.g., right, straight, acute, obtuse); construct right, straight, acute, exterior and obtuse angles using ruler and pencil and/or software; estimate the magnitude of various angles (using 45 and 90 referents); measure selected angles to the nearest degree using a protractor; draw angles using a protractor and classify them by magnitude (e.g., angles corresponding to a complete turn – i.e., 90, 180, 270, 360 degrees); measure angles between intersecting lines (to verify opposite angles rule); construct congruent angles and angle bisectors with protractor and pencil; measure interior angles in a triangle (angle sum theorem); measure exterior angles in a triangle

Geometry (cont)

Geometric Shapes and Objects (cont)

5.1.1

Constructions (cont)

5.1.1.2 plane figures

identify geometric/non-geometric properties in the environment; (e.g., # of edges of a building vs. different colours of buildings); construct various plane figures (e.g., squares, triangles, circles); recognize similarities/differences between plane shapes (e.g., square, triangle, rectangle, rhombus, circle); construct and classify triangles (e.g., equilateral, isosceles, scalene, right); construct various quadrilaterals/polygons; classify attributes of various quadrilaterals/polygons (e.g., rectangles, parallelograms, kites, trapezoids, rhombus) by properties (i.e., # diagonals, # faces, # vertices); label plane figures using appropriate conventions (e.g., triangle ABC); construct parallelograms from various sources of information (e.g., instructions, conditions, drawing templates); construct hexagons from various sources of information; construct circles, chords, diameters and radii using pencil, ruler and compass; describe properties of a circle (i.e., centre point, diameter, radius); construct attributes of plane figures (i.e., diagonal, altitude, hypotenuse, perpendicular bisector, apothem, radius, diameter and chord where appropriate); construct complex plane figures by combining simpler forms, compute perimeter/area of various plane figures; evaluate the adequacy of constructed figures based on known properties

Geometry (cont)

Geometric Shapes and Objects (cont)

Constructions (cont)

5.1.1.3 objects

recognize various geometric objects or parts of objects in your everyday life; characterize and classify geometric objects based on edges, vertices and faces (e.g., cube, parallelepiped, sphere, cone); construct various 3-D objects (using concrete materials and/or software); construct 2-D nets of convex polyhedrons (e.g., cube, prism, pyramid) corresponding to 3-D objects (and vice versa); classify properties of convex polyhedrons (e.g., cubes, prisms and pyramids) based on geometric properties (i.e., faces, vertices and edges); identify objects placed in different orientations on the basis of their geometric properties; identify various 2-D plane figures on surfaces of 3-D objects (e.g., circle on a cylinder, triangle on a pyramid); verify Euler's theorem of convex polyhedrons; represent 3-D objects and perspectives using orthogonal projections (and parallel and central projections); compute surface area of various cubes and parallelepipeds given corresponding nets

5.1.1.4 symmetry and congruence

identify lines of symmetry (i.e., diagonals, altitudes) through various 2-D plane figures/3-D objects placed in different orientations; construct line(s) of symmetry through 2-D plane figures using software; use principles of symmetry to construct complex symmetrical plane figures; construct tessellations and frieze patterns by applying various geometric transformations to 2-D plane figures; construct various congruent 2-D plane figures given partial information about sides, diagonals and angles; use principles of congruence too describe 2-D plane figures that have been transformed; construct images of 2-D plane figures following one or more transformations (and after dilatations with positive scale factors)

Geometry (cont)

5.1

Geometric Shapes and Objects (cont)

Properties

5.1.2.1 lines and angles

identify straight line segments in rectangles and squares; identify parallel and perpendicular lines in selected plane figures; use diagonals in rectangles and squares to identify various triangles; solve problems using opposite and complementary angles rules; relate properties of geometric parallelism to perspective and architectural drawings in the environment; solve problems using parallel line, alternate angles, and corresponding angles theorems; classify properties of selected angles (e.g., straight, right, acute, obtuse); use sum of angles in triangles theorem to solve problems; use sum of angles in triangles theorem to determine angle sums in various polygons (e.g., sum of interior angles in a quadrilateral); apply line bisector-, angle bisector-, parallel lines-, and perpendicular lines theorems to solve problems in context

5

Geometry (cont)

Geometric Shapes and Objects (cont)

5.1.2

Properties (cont)

5.1.2.2 plane figures

describe geometric properties of triangles (i.e., # sides, # vertices); classify equilateral, isosceles, scalene, right, acute and obtuse triangles by magnitudes of interior angles; describe Pythagorean theory as it applies to right triangles; compute lengths of sides in right triangles using Pythagorean theory; compute basic trigonometric ratios (sine, cosine, tangent) from right triangles; use exact values of selected angles (e.g., $\sin 45^\circ$, $\cos 60^\circ$) to solve problems; apply sine and cosine rules to solve problems in context; use the triangle area formula ($A = \frac{1}{2}ab \sin C$) to find unknown sides or angles in any triangle; classify selected quadrilaterals with parallel/non-parallel sides; interpret cultural and artistic designs based on diagonals and quadrilaterals; describe geometric attributes of quadrilaterals (i.e., # sides, # vertices); use differences/similarities between triangle and quadrilateral properties to solve problems in context; classify parallelograms (i.e., by lengths of sides); classify geometric figures as regular and irregular polygons; describe properties of convex and non-convex polygons; compose/decompose composite figures into simpler 2-D figures; describe properties of polygons with up to 100 sides; use sum of angles in pentagons and hexagons to solve problems; compute perimeter/area of triangles, parallelograms and trapezoids; describe properties of circles (i.e., centre, radius, diameter, circumference, chord, tangent, arc, sector); compute circumference, radius and diameter in selected circles; describe the concept of locus; estimate circumference of a circle (i.e., $C \doteq 3.14 \times D$) and use this to estimate the value of π ; use geometric relationships between radius, circumference and diameter to solve problems; compute perimeter of regular and irregular polygons; compute area of a circle using the formula; analyze assertions about properties of various plane figures and judge their validity; identify various images of geometric figures in the environment; describe meanings of triangles, quadrilaterals and parallelograms in indigenous cultures; interpret the use of regular and irregular

Geometry (cont)

Geometric Shapes and Objects (cont)

Properties (cont)

- 5.1.2.3 objects
- 5.1.2.4 congruence and similarity
- 5.1.2.5 symmetry

identify images of 3-D objects observed in the environment; compare various 3-D objects using non-standard attributes (e.g., long vs. short, wide vs. narrow); classify 3-D objects by geometric attributes (e.g., # and type of lateral and basal faces, # total faces, # edges, # vertices); use relationships between 2-D plane figures and 3-D objects to fold and unfold 3-D objects from nets; compute height of selected 3-D objects; derive surface area formula for regular prisms; compute surface area of prisms, cubes, cylinders, pyramids and cones; derive volume formula for regular prism; compute volume of prisms, cubes, cylinders, pyramids and cones; compute surface area/volume of spheres, compute surface area/volume of rectangular prisms and cylinders

identify properties of congruent plane figures (i.e., side and angle correspondence) in frieze patterns and tessellations; identify congruent line segments and angles in 2-D plane figures; use triangle congruence rules to solve problems; identify criteria for congruence between quadrilaterals and contrast this with triangle congruence; use congruence properties to classify 3-D objects (e.g., # faces completing the base of prism vs. pyramid); identify similarity rules for selected 2-D plane figures (e.g., triangles); use properties of similarity and congruence to solve problems; use 2-D plane figure transformations to link figures and their images; determine invariant characteristics of similar and congruent figures (e.g., triangle similarity vs. triangle congruence); justify statements about congruence and similarity

identify axes of symmetry in selected 2-D plane figure; identify symmetrical properties of 3-D objects (e.g., cuboid, prism, sphere); use symmetry properties of 2-D plane figures to elaborate and extend diagrams and artwork; use symmetry properties of 2-D plane figures to investigate Mayan cultural symbols and signs

Geometry (cont)

5.2

Position and
Direction

5.2.0

Position and
Direction

5.2.0.1

translations,
rotations,
reflections and
dilatations

describe positional change with respect to a fixed point of reference; use instructions to create movement on a coordinate grid (e.g., move two squares to the north); describe relative location and paths of motion; copy and elaborate geometric patterns observed in the environment; translate selected figures using various materials (paper and pencil, software); rotate/reflect selected plane figures using various materials; represent points and 2-D plane figures resulting from a series of transformations; recognize 2-D plane figures observed in the environment that have been transformed; relate geometric transformation properties and 2-D plane figure congruency (using software); relate translation/dilatation to interpret direction of travel or displacement and to proportionality of shrinking and growing figures; use scale factors (including fractional and negative scale factors) in dilatations; relate tessellations to geometric transformation; relate vectors to geometric translation; locate and describe objects in space and/or their projections using different orientational referents (e.g., top, bottom); measure/estimate lengths of geometric elements found in the environment and in cultural patterns; recognize equivalencies between cardinal points and the Mayan Cross

Geometry (cont)

5.3 Properties of Space

5.3.0 Properties of Space

5.3.0.1 Cartesian plane

describe position of everyday objects in the environment using common language (e.g., right, left, up, down); locate objects relative to other objects using common language (e.g., inside, at the edge); describe relative/absolute position of geometric 2-D plane figures on a simple map or grid using common language (e.g., two units to the left, four units north of the origin); use ordered pairs to locate points on a grid; identify horizontal/vertical lines in the first quadrant of the Cartesian plane; plot points and construct simple shapes in the first quadrant; use tables of values to locate and construct vertices of 2-D plane figures; investigate properties of lines and plane figures by plotting points on a coordinate graph; determine ordered pairs associated with points of intersection between lines (perpendicular or not); devise algebraic rules relating elements of ordered pairs to create patterns; describe properties of linear relations by plotting ordered pairs; compute slopes of linear relations and represent slope graphically; describe location of objects in 3-D coordinate space; locate 2-D projections of 3-D objects on a coordinate plane; interpret and elaborate spatial relationships between plane figures and 3-D objects; interpret and elaborate representations of perspective in space and explain how proportionality is involved; use ordered pairs to locate positive/negative numbers on the Cartesian plane; use positive/negative integers to plot problem situations encountered in everyday life (e.g., temperature, elevation); make inferences from graphs (i.e., interpolate, extrapolate) to solve problems; describe limits of graphs when used to interpret data

Algebra

6.1	<h2>Non-Numerical Patterns</h2>	6.1.1 Relations	6.1.1.1 patterns in the environment	<p>create and describe increasing/decreasing patterns using concrete objects; construct patterns (e.g., colours, shapes, sounds, geometric patterns) from everyday life; relate properties of non-numeric patterns to geometric transformations and symmetry; relate non-numeric patterns to mosaics, tessellations and frieze patterns</p>
			6.1.1.2 patterns in culture	<p>recognize cultural patterns through observations of traditional community handicrafts, dance, music, artwork, and theatre; copy and elaborate patterns observed in traditional cultural images (e.g., mosaics, symbols)</p>
6.2	<h2>Numerical Patterns</h2>	6.2.1 Relations	6.2.1.1 sequences	<p>create patterns using concrete, pictorial and symbolic means; express number patterns as sequences following a given rule (including combinations of up to two arithmetic operations); describe numerical patterns derived from geometric figures (e.g., triangles, quadrilaterals); determine new terms in a sequence when at least three terms are given; describe roles of unknowns, variables and constants in sequences; formulate numeric rules to describe a sequence; determine sets of ordered pairs satisfying a given relation; elaborate a sequence based on a deduced pattern or rule; devise a formula for the nth step of a process; determine terms and term numbers in linear sequences; represent linear growing/shrinking patterns symbolically and graphically; describe numeric rules by comparing sets and mapping the domain of one set to the co-domain of the other; determine the range of a set given a numeric rule; solve 3x3 magic squares</p>

Algebra (cont)

Numerical Patterns (cont)

Functions

Relations (cont)

Linear functions

6.2.1.2

6.3.1.1

properties of algebraic expressions

properties

describe the nature of coefficients, terms, constant terms and like terms; construct algebraic expressions using a register or machine metaphor; represent algebraic expressions given specific language-based instructions and vice versa; construct equivalent algebraic expressions given a symbolic description of a rule (e.g., $t = 2a + 4$; $2b = 4a + 8$); describe equality and inequality in terms of equilibrium and imbalance (using concrete materials); use appropriate algebraic symbols to express equality and inequality (i.e., $<$, $>$, $=$); construct inequalities (e.g., $4 < a - 3$); solve algebraic equations (in one variable); describe patterns of arithmetic sequences using a table of values

translate problem situations in language to symbolic mathematical relationships; describe algebraic operations pictorially and symbolically (e.g., relating them to area or volume); transform algebraic expressions to equivalent expressions through manipulation (e.g., $y = 4t + 2$; $y = 2(t + 1)$); isolate and simplify terms in simple functions (e.g., $t = 1 + 2m$; $m = (t - 1)/2$); solve one-step equations in one unknown (involving addition/subtraction); express one-step equations with one unknown using various representational forms (e.g., graph, table, bar chart); use formulas to depict continuous processes; compute the value of a given variable in a linear expression; determine the inverse of a function; recognize succession of two functions as a composite function; check results using various strategies; justify steps in solving equalities and inequalities using appropriate mathematical language and symbols

6

Algebra (cont)

6.3

Functions (cont)

6.3.1

Linear functions (cont)

6.3.1.3 linear equations

6.3.1.2 algebraic notation

recognize and use appropriate notation for mathematical statements (e.g., ab in place of $a \times b$; $3y$ in place of $y + y + y$ or $3 \times y$); substitute numerical values into formulas and symbolic expressions to solve problems for specific conditions; simplify algebraic expressions (e.g., collecting like terms, expanding across brackets, common factoring, manipulating algebraic expressions across equal- and inequality-signs); relate linear algebraic expressions to simple translations and reflections by plotting points on the Cartesian plane

describe linear change in various ways (e.g., tables of values, machine metaphor); describe relationships between tables of values and graphs to explain continuous and discrete data; use linear functions to depict uniform growth/decay; solve first-degree equations in one unknown with various strategies (e.g., trial-and-error, drawing depicting change); compute slope and intercepts from plots of linear functions; interpret slope and area under graphs of linear functions (e.g., distance-time graphs); model contextual situations described in language with linear equations; solve linear equations graphically; solve linear equations with rational coefficients using graphic and symbolic means; describe related functions (e.g., generalizing them as sums of constants representing a family of linear functions); solve linear equations in two variables (using standard form $y = mx + b$); use linear equations in standard form to identify parallel and perpendicular lines; derive the equation of a line given two points or one point and a slope; solve linear inequalities in one- and two-variables using algebraic and graphical means; check results of a solution attempt using back-substitution; interpret solutions and make decisions based on contextual information

Algebra (cont)

Functions (cont)

6.3.2

Non-linear functions

6.3.2.1 properties

determine terms and term numbers of selected geometric sequences; simplify selected binomial and trinomial algebraic expressions (including those with surds) by common factoring/difference of squares factoring/ trinomial factoring; derive expressions for the nth term in a quadratic sequence; compute the roots of a quadratic equation using algebraic means; identify and interpret roots, intercepts and turning points of selected quadratic equations using graphical means; plot selected translations and reflections of non-linear functions; determine intercepts, slopes and areas under graphs of non-linear graphs (e.g., distance-time graphs, velocity-time graphs); interpret graphs of simple cubics, reciprocal functions, exponential functions and trigonometric functions

6.3.2.2

simultaneous equations

solve systems of two simultaneous equations in two variables (linear/quadratic) algebraically; approximately solve systems of two simultaneous equations in two two variables (linear/quadratic) graphically

6.4.0.1

properties

determine length and bearing of a vector from a given point; use vector components along the number line to solve problems; identify properties of the zero vector; identify equal vectors; derive geometric arguments and proofs using vector constructions

6.4.0.2

arithmetic operations

add two vectors in component form (i.e., $AC = AB + BC$); apply vector arithmetic to solve selected problems involving vectors and scalars

Linear functions (cont)

6.3.1.4

simultaneous equations

solve systems of two simultaneous linear equations, each with two variables, using algebraic means; determine approximate solutions for systems of two simultaneous equations using graphical means

6.4 Vectors

6.4.0

Vectors

Algebra (cont)

6.5 Variation

6.5.0 Variation

6.5.0.1 ratio

demonstrate that one quantity can be expressed as a fraction of another (using concrete materials; e.g., 2 red apples and 3 green apples can be expressed as $2/3$); relate ratio to proper fractions and determine the simplest form; relate ratio to multiplicative relationships between two quantities (e.g., $2 \times \frac{1}{2} = 2/2 = 1:1$); relate ratio to linear functions; relate ratio to perimeter, area and volume relationships; relate ratio to scale factors in dilatations; use ratios to convert between compound units (e.g, speed, rates of pay, price per unit, density, pressure)

6.5.0.2 percentage

define percentage as the number of parts per hundred; apply percentage and percentage change in fractional and decimal forms; relate percentage to simple ratios (i.e., one quantity as a percentage of another related quantity); compute percentages to one decimal point; solve initial value problems involving percentages; solve simple interest problems involving percentages and interpret results given context and available information

6.5.0.3 proportion

relate equivalent ratios to the concept of proportion; relate numeric and geometric patterns to proportion (e.g., linear sequences, similar triangles); solve simple proportion problems; interpret and apply direct proportion in specific situations; interpret and apply inverse proportion in specific situations; use various strategies to compute direct and inverse proportion (e.g., scales, dilatations, velocities) using various representational sources (e.g., tables, constants of proportionality); relate direct and inverse proportion to slope using algebraic and graphical means; solve proportionality problems involving non-linear function (e.g., growth and decay, compound interest)