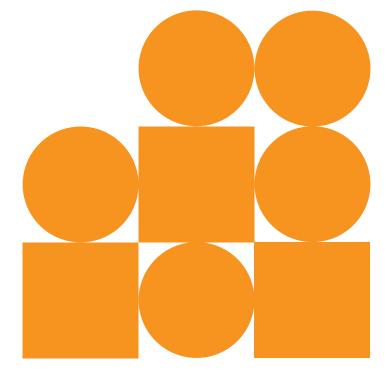
Educational Practices Series

Math Anxiety

by Denes Szűcs and Irene C. Mammarella







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In 1969, the IBE became an integral part of UNESCO, while retaining intellectual and functional autonomy.

The IBE is a UNESCO category I institute and a center of excellence in curriculum and related matters. Its mission is to strengthen the capacities of Member States to design, develop, and implement curricula that ensure the equity, quality, development-relevance and resource efficiency of education and learning systems.

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About the Series

The Series was started in 2002, as a joint venture between the International Academy of Education (IAE) and the International Bureau of Education (IBE). So far 30 booklets have been published in English and many of them have been translated in several other languages. The success of the Series shows that the booklets meet a need for practically relevant research-based information in education.

The series is also a result of the IBE's efforts to establish a global partnership that recognizes the role of knowledge brokerage as a key mechanism for improving the substantive access of policymakers and diverse practitioners to cutting-edge knowledge. Increased access to relevant knowledge can also inform education practitioners, policymakers and governments how this knowledge can help address urgent international concerns, including but not limited to curriculum, teaching, learning, assessment, migration, conflict, employment and equitable development.

Governments need to ensure that their education systems meet their core and indisputable mandate, which is to promote learning and, ultimately, to produce effective lifelong learners. With the aggressive pace of contextual change in 21st century, lifelong learning is a critical source of adaptability, agility to adapt, and the resilience required to meet challenges and opportunities. Yet, for many countries around the world, effective facilitation of learning remains a daunting challenge. Learning outcomes remain poor and inequitable. Intolerably high proportions of learners fail to acquire prerequisite competences for lifelong learning such as sustainable literacy, digital literacy, critical thinking, communication, problem solving, as well as competences for employability and for life. Systems' failure to facilitate learning co-exists with impressive advancements in education research, driven by research from diverse fields, including the sciences of learning, particularly the neuroscience of learning, and advancements in technology.

The IBE's knowledge brokerage initiative seeks to close the gap between scientific knowledge on learning and its application in education policies and practice. It is driven by the conviction that a deeper understanding of learning should improve teaching, learning, assessment, and policies on lifelong learning. To effectively envision and guide required improvements, policymakers and practitioners must be fully cognizant of the momentous dialogue with research.

The IBE recognizes the advancements already made, but also that there is still much more work to be done. This can only be achieved through solid partnerships and a collaborative commitment to building on previous lessons learned and continued knowledge sharing.

The Educational Practices booklets are illustrative of these ongoing efforts, by both the International Academy of Education and the International Bureau of Education, to inform education policymakers and practitioners on the latest research, so they can better make decisions and interventions related to curriculum development, teaching, learning and assessment.

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Introduction

Many students have a debilitating emotional reaction to mathematics termed "mathematics anxiety".

Mathematics is often perceived as a difficult subject by many students, parents and teachers alike. Difficulties with the subject are most often attributed to cognitive factors (lack of ability, preparedness, practice and knowledge). Emotional factors are often overlooked and are easily written off as potential persistent and serious causes of mathematical learning difficulties. However, it is increasingly recognized in psychology and education that several students have serious negative emotional reactions to mathematics. These emotional problems can lead to performance difficulties and/or can become obstacles that discourage students from further mathematics training even if their performance is good.

This debilitating emotional reaction to mathematics is termed "mathematics anxiety" (MA). MA is "a feeling of tension and anxiety that interferes with the manipulation of numbers and the solving of mathematical problems in... ordinary life and academic situations" (Richardson and Suinn, 1972).

MA ranges from the feeling of mild tension to experiencing strong fear. MA is not restricted to classroom situations or to children. Rather, MA can generalize to out of school situations and can affect adults. For example, MA can manifest itself in everyday situations about handling numbers such as when counting change in shops or doing basic mathematics under time pressure. MA is often present in normally performing students, discouraging them from choosing math related careers. A structured universal prevention program for MA does not yet exist. Most of the activities we suggest here are based on the principles of Cognitive Behavioural Therapy (CBT) and the Rational Emotive Behaviour Therapy (REBT). These methods help individuals to identify self-defeating thoughts and feelings, challenge the rationality of those feelings, and replace them with more productive beliefs.

Suggested readings: Ashcraft, 2003; Dowker, 2005; Mammarella, Caviola, and Dowker, 2019; Richardson and Suinn, 1972.

How do I know that my students have Math Anxiety?

Teachers probably have their own valuable impressions about the MA of their students. Quantitative questionnaires can help teachers to compare the MA levels of students along standard criteria.

Research findings

- In educational and psychological research academic anxieties are typically determined by questionnaires (notably, academic anxieties are not considered clinical anxiety disorders).

 The Abbreviated Math Anxiety Scale (AMAS), a 9-item adult questionnaire, has been modified for use with 8-13 year-old children. This modified AMAS (mAMAS) is freely available online and can be used without specific permission (see Supplementary Material in Carey, Hill, Devine, and Szűcs, 2017). Each of nine questions yields a score between 0 and 5. Higher score means higher level of MA. The mAMAS has two subscales, one focused on anxieties related to mathematics learning, the other related to anxieties related to mathematics testing (evaluation). The scales are highly correlated but can be considered to measure different aspects of the same MA construct.
- It is reasonable to consider children scoring in the top 10% on the mAMAS to have high levels of MA.

 Test score boundaries are set arbitrarily, and they also depend on the cultural environment. For example, in some countries it may be easier or harder for girls and boys to acknowledge their anxieties than in other countries. Hence, before large scale use in a country it is recommended to collect baseline score data from hundreds or thousands of children. This data will enable the users to determine what score threshold characterizes the top 10% most anxious children in a culture. Without such validation work individual scores should be interpreted cautiously. Validation work is typically done by local universities and research institutions.

In the classroom

- Using mAMAS teachers can measure overall MA and two aspects of MA
- The odd (3, 5, 7 and 9) questions of mAMAS except question 1 measure math learning anxiety, meaning that students feel nervous and worried about learning new math concepts. The even questions measure math evaluation anxiety. This means that students feel anxious about a math test or homework that will be used to evaluate their performance.
- · Questionnaires are not available in many countries and using multiple sources of information about MA is helpful in any case. Teacher impressions about the manifestations and causes of MA are important. Teachers can explore these by chatting with their students and observing student behaviour. Anxiety often becomes stronger when math problems need to be solved quickly or in front of the class. If a student regularly fails in these situations or avoids these situations, teachers should devote more attention to the student's emotions about math. Teachers can ask parents whether the child sleeps well before math tests or has stomach- or head-aches before going to school, especially before math classes. Some students feel comfortable working on their own but not in front of the class. This may show up as discrepancy between good homework performance versus poorer performance in front of peers. Such profile may suggest that the student has some degree of MA.
 - If the teacher has the impression that a student has high MA, she/he could suggest a systematic assessment of MA to parents. Such assessment is usually carried out by school psychologists who are trained to perform high-quality assessment.
- Questionnaire responses represent personal data protected by law and should not be shared with anyone except as allowed by regional legal regulations.

Math Anxiety is distinct from other forms of anxiety and becomes more distinct from general anxiety by secondary school.

It is important to distinguish between general anxiety for everyday events, test anxiety for any test situation and math anxiety.

Research findings

- Math anxiety, general anxiety and test anxiety are distinct forms of anxiety.
 - General anxiety refers to an individual's tendency to feel anxious about everyday situations. For example, people with general anxiety may be worried about their health, their family and also about making simple decisions about their daily life. Test anxiety is anxiety for all kinds of test situations, not only towards mathematics. Children with high general anxiety and high test anxiety also tend to have high math anxiety. However, the three anxiety forms are distinct.
- Academic (test and math anxiety) and non-academic (general) anxieties become more and more separated from primary to secondary school.
 - In both primary and secondary school the higher the MA the worst the math performance. In 8-9 year-old primary school pupils, general anxiety, test anxiety, MA and their relation to math performance are very similar: if a child shows high general anxiety their test and math anxiety are likely also high. The higher all these anxiety levels the worse the math scores. However, the picture is different in 11-13 year-old secondary school children. At this age some children show high general anxiety levels but low test and math anxiety levels whereas other children show high math and test anxiety levels but relatively low general anxiety levels. Math scores are the lowest in the second group of children. This suggests that general anxiety and academic anxieties are more differentiated by secondary school.
- Math anxiety develops in different ways in different children.
 Some children start primary school with a high level of general anxiety. By secondary school some of these children will still have high general anxiety but not necessarily academic anxieties. Yet, other children with initial high general anxiety may develop more specific academic anxieties while ultimately feeling less anxious about everyday events.

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In the classroom

- It is important to distinguish between MA, test anxiety and general anxiety.
 - Although the above anxiety forms are related, this does not mean that students with high MA also have high general anxiety and vice versa. The relation among these forms of anxiety could be interpreted in terms of risk factors: the presence of one type of anxiety (for example, general anxiety) can increase the probability that students develop other related forms of anxiety (test anxiety or MA).
- Preventing high general anxiety by early school interventions is crucial in educational settings.
 - Teachers could start to talk about emotions as early as possible in school with their students. Teachers could ask students to list threatening situations in school. The nature of these situations could be discussed and if many students list a situation as threatening, a school psychologist may carry out more systematic assessment.
- It is important for teachers to understand the intensity of students' anxiety.
 - Students should be aware that moderate levels of anxiety can be positive and may boost performance. Anxiety becomes a negative force when it is felt at extreme levels and leads to freezing or running (avoidance). Students freeze when overwhelmed by many negative and irrelevant thoughts that impede them from performing a task. Students avoid a task for the fear of not being able to cope with it.

Suggested readings: Hill, Mammarella, Devine, Caviola, Passolunghi, and Szűcs, 2016; Carey, Devine, Hill, and Szűcs, 2017.

Math Anxiety leads to task-irrelevant thoughts that negatively interfere with mental resources needed for problem solving.

Teachers should help students to understand that their anxious task-irrelevant thoughts can negatively affect performance.

Research findings

- The higher is the MA of pupils the worse is their math performance. The Program for International Student Assessment (PISA) tested 15-year-olds' academic achievement worldwide in 2012. In 63 of the 64 education systems tested the higher the MA the lower the math achievement. Notably, this sample level association does not mean that all students with high MA are very poor achievers (see Section 7).
- Higher MA is associated with poorer math performance because children tend to attend to their own worries rather than devote their energies to problem solving.
 Due to MA children may be preoccupied by their perceived inability to solve the mathematics task at hand, they may think about future parental punishment if they achieve bad results or, they may anticipate that their peers will react negatively to them. These worries can occupy the so-called working memory of children, not leaving adequate memory capacity to solve mathematics problems at hand (working memory is the mental capacity enabling us to hold and manipulate information in our mind).
 - More difficult mathematics tasks generally require more working memory capacity than easier tasks. Hence, the working memory disruption caused by MA has more potential to negatively impact performance on more difficult rather than easier math problems.

In the classroom

 Teachers, should help students to become aware of how their taskirrelevant thoughts can affect their achievement.
 When solving math problems students with high MA will anticipate that something negative will happen (lack of success; too difficult task; classmates making fun of them, etc.). Being aware of the negative impact of these thoughts can help students to understand that decreased performance is not due to their lack of ability.

- Teachers should give children the opportunity to talk about their emotions, and the thoughts associated with anxious situations.
 Considering the impact of negative thoughts and emotions, time devoted to their discussion is well spent and probably pays off in terms of improved performance due to improved meta-cognitive awareness of children.
- Teachers could help students to understand that making errors
 during learning and mathematical experimentation is completely
 natural and that errors can even help future understanding.
 Looking at their performance in this context may help to improve
 student self-competence. Recognizing the "puzzle-solving aspects"
 of mathematics may motivate students' interest towards the
 discipline.
- Teachers could help older students to become aware of the relationship between thoughts, emotions and behaviour. Such awareness can help students to get rid of their negative thoughts. Each student could choose a sentence to use when negative thoughts became too intense (for example, "stop thinking", "think positive", "breathe deeply"). However, this method may not work with younger students (ages 6-8) because their meta-cognition is still underdeveloped.
 - Teachers can ask students to work in small groups and write down their thoughts about difficult situations that they experience in school. It may be useful to involve teachers who do not teach math in this activity so that students would feel free to express their thoughts. Teachers could give examples of the most "useful" and positive thoughts related to a situation. An example of a "useful" thought could be "I am worried about the math test, but I studied hard this time, and if I stay calm, I can succeed". Students could then be asked to find a "useful thought" for every "bad thought" and to write down the positive thoughts.

Girls often show higher levels of Math Anxiety than boys even when the math achievement of girls and boys is not different

MA may be higher in girls than in boys because of gender stereotypes about mathematics, more anxious predisposition in girls and girls' more accurate reporting of MA.

Research findings

- Girls often show higher MA than boys in many cultures.
 The gender gap in MA is already present in grade 2-3 of primary school. In contrast, girls and boys at these grade levels typically achieve at the same level in mathematics. Hence, it is unlikely that differences in objective indicators of math performance result in higher MA in girls than in boys.
- Mathematics and science ability and suitability related gender stereotypes are prevalent in many societies. Science and mathematics are often considered male domains leading to strong gender stereotypes. Experiencing stereotype threat is an important factor in inducing MA in girls. Stereotype threat is when a person is in such a situation that she/he feels to be at risk of confirming negative stereotypes about their social group. For example, a stereotype may be that "girls do not understand math well". Girls' test performance may be negatively affected if such gender stereotype is expressed before a math test is taken (e.g., by showing a video of a girl struggling with a math problem).
- Girls often report lower levels of self-confidence and self-efficacy in
 mathematics than boys.
 Self-confidence refers to the trust in one's ability to achieve a goal.
 Self-efficacy is defined as a personal judgment of how well one can
 deal with prospective situations. Students with high self-efficacy
 will exert sustained effort targeting successful outcomes. Low
 mathematics self-efficacy and mathematics self-confidence are
 linked to high MA.
- Girls are generally more anxious than boys and they may also report their MA more accurately than boys.
 Besides higher MA girls typically also report higher general anxiety and test anxiety than boys. Their higher anxiety levels may predispose them to develop higher MA than boys. Further, girls

have higher levels of meta-cognitive readiness than boys of the same age. Therefore, girls may be able to report their own anxiety perceptions more accurately than boys. It may also be culturally more acceptable for girls than for boys to admit their own anxieties. In many cultures boys are expected to suppress their emotions more than girls. Notably, if girls perceive themselves anxious, that perception may strongly impact their performance.

In the classroom

- A mixed gender group discussion about coping with MA could be particularly useful for girls.
 During discussions girls could recognize the different ways that
 - During discussions girls could recognize the different ways that their male classmates deal with MA. During discussions about this topic, teachers could reinforce well-functioning coping strategies with MA without reference to gender differences.
- Teachers should understand their own gender ability beliefs towards mathematics.
 - Sometimes teachers unintentionally reinforce traditional ideas about what girls and boys can study. This limits girls' potential in mathematics. Teachers could form groups to discuss their own gender stereotypes about mathematics.
- Teachers should avoid attributing performance outcomes to children's gender.
 - In contrast, teachers should give individual (gender neutral) performance assessments. Existing gender stereotypes about math learning can be fought by real-life examples of counterstereotypes (for example, "My daughter is studying at the faculty of mathematics."; "His sister has a degree in software engineering."). If a girl attributes her own weak performance to the fact of being female, teachers could take the chance to discuss stereotypes and their negative effects with the whole class.
- Schools should promote STEM subjects to girls and boys equally.
 Teachers should encourage all students to break down gender
 stereotypes about mathematics learning. It is important to
 remember that after parents, teachers are probably the most
 important reference figures for children and their beliefs and
 expectations strongly condition children's behaviour and
 performance.

Suggested readings: Beilock, Gunderson, Ramirez, and Levine, 2010; Dweck, 2007; Zirk, Lamptey, Devine, Haggard, and Szűcs, 2013.

A vicious circle may operate between perceived poor math performance and Math Anxiety

Gradual confidence building is crucial in fighting MA.

Research findings

- Relatively weak math performance and high MA can interact. Some assume that poor performance leads to high MA. Others think that high MA leads to poor performance. These views are not mutually exclusive. First, the two causal pathways may be true for different children. Second, MA and perceived poor math performance may form a vicious circle: Some children may first be convinced that they cannot understand math. This belief may lead to MA and avoidance of math tuition and elective math classes. Avoidance of tuition can lead to relatively poor performance (under parental, teacher or child expectations) justifying and amplifying MA. Increased MA can lead to further dislike of math and so on... It is important to realize that what primarily matters is the child's subjectively perceived performance level. Even well performing students can compare themselves to the top of the class, or can have unrealistic expectations.
- Children with cognitive math learning difficulties have about double the chance of having high MA than other children. Vicious circles may be more likely to develop in children with specific math learning difficulty (MLD) who may get extreme negative feedback to their performance. Indeed, while by our definition the 10% of children getting the highest scores on the MA scale have high MA, 22% of the subgroup of children with MLD have high MA.

In the classroom

 Teachers should aim to understand the beliefs of students towards mathematics.

Student beliefs can be modified step-by-step to bring them in line with increasing performance goals. Teachers could generate a list of their students' beliefs about mathematics learning. This list will likely include several false beliefs (e.g., if you do not find the right response to a math problem in few minutes, you will never solve that problem). Teachers can discuss responses with students and give them real-world examples demonstrating that some beliefs are false. This activity may start improving student self-competence

- (self-competence refers to perceived ability in academic areas).
- Feedback should acknowledge invested effort and improvement rather than simply comparing student performance to reference children or the class.
 - Effort and dedication should be always lauded even when solutions fail or are not perfect. Feedback should emphasize the new concepts learnt and effort invested rather than performance. Feedback should focus on the process of solution and not only on the correct/incorrect status of final answers. Of course, it is important to get the correct response to a math problem. However, in order to improve self-competence, students should be helped to understand how far away they are from the correct solution, and how much effort they still need to dedicate to reach this solution.
- Assignments for students with MLD should involve at least some
 tasks that they can solve correctly.
 The self-efficacy and self-confidence of MLD students can increase
 based on their individual perception of being able to deal with
 specific tasks. Performance goals should be increased in as small
 steps as possible. This will allow a gradual build-up of selfconfidence and self-efficacy in mathematics. If children consistently
 fail on some problems teachers should make an effort to link these
 problems to other, already well solved problems.
- Time pressure should be avoided as much as possible in the case of students with MLD and high MA.
 Avoiding time pressure will at least decrease MA levels freeing up mental resources to focus on the task at hand.
- Concrete examples could be given when new mathematical
 concepts are presented.
 Students can be asked to discover different strategies to find the
 right answer. If one solution method is more preferable in class
 than another one (for example, because it is faster) then it should
 be ascertained that students understand these alternatives and why
 a particular method is to be preferred.

Suggested readings: Carey, Hill, Devine, and Szűcs, 2016; Mammarella, Donolato, Caviola, and Giofrè, 2018; Devine, Hill, Carey, and Szűcs, 2018.

Emotional and cognitive mathematics problems are distinct

Different interventions are needed to address mathematical learning problems of cognitive vs. emotional origin.

Research findings

- Approximately 80% of children with high MA are high achievers. It is a frequent misconception that only very poor achievers show high levels of MA. However, about 80% of children with high MA are high math achievers. It is crucial to deal with the MA and potential emotional difficulties of these children. Notably, while moderate levels of anxiety can improve performance, the above children experience high rather than moderate levels of MA. Such high MA levels are more likely to have some negative rather than positive consequences. Indeed, the most notable impact of high MA is the lack of very high performing pupils at very high levels of MA rather than having many very poorly performing pupils at very high levels of MA. We can speculate that MA may preclude normal ability children from fulfilling their potential in mathematics.
- About 80% of low achieving children do not have high MA. 80% of children with mathematics learning difficulty (MLD) or "developmental dyscalculia" (DD) do not have high MA (see Section 6). This may mean that poorly performing children have not internalized values related to good/poor math performance and/or children will not anticipate any negative/positive reaction of parents related to their math performance. It is also possible that some children with poor math performance simply lack the meta-cognitive abilities to self-reflect.
- High achievers with MA may be most in danger of opting out of
 elective math courses.
 Due to their high level of MA these otherwise competent
 children may bit-by-bit opt out from elective math education
 opportunities; they may just choose to do as little mathematics as
 they can get by (depending on parental and teacher expectations)
 and will likely not venture on math-related career opportunities
 even if they would prefer these.

In the classroom

- Teachers should watch out for signs of high MA in students who
 perform well academically in mathematics. For example, some
 well performing children may stubbornly reject further math
 education opportunities. This may be a sign of high MA (obviously,
 some children may really just prefer alternative career options to
 mathematics related ones).
- Teachers should devote some effort to understanding how students value math.
 - Valuations and student performance self-assessments could be collected before and after exams (e.g., by questionnaires) and compared anonymously with actual exam results. In students with average to good performance it is possible that the higher value a student attaches to mathematics the higher is his/her MA (math tests represent a higher stakes activity for these students). In contrast, low performers may not attach much value to math. Anonymized valuations, self-assessments and their relation to actual results could be discussed in class. Overall, competence beliefs may moderate the influence of MA. Thus, group discussions can be useful for generating different points of views and can be indirectly insightful and helpful for students showing a discrepancy between their competence beliefs and their actual performance.
- When possible, math exercises should relate to real world situations.
 - Particularly for weaker students it is important to see that mathematics can be fun and useful. In smaller children playful games that demonstrate/require the use of mathematics in everyday situations may help alleviate MA. Homework and assignments should aim to reproduce real life situations. Teachers could also ask their students which occupations they would like to do in their life and ask students to discuss in what ways mathematics could be helpful in succeeding in their job choices. This may help to improve the intrinsic motivation of students to learn mathematics.

Interpretation of experiences is crucial in the origins of Math Anxiety

Students with low and high MA experience similar events in the school but they interpret them differently.

Research findings

- The interpretation of school experiences differed between students with low and high MA.
 - An interview study found that many primary school students with high MA thought that the required work in math was beyond their capabilities. Close to half of students with high MA were afraid of being asked maths questions in front of a class but most students with low MA did not have this experience. MA was triggered when comparing work unfavourably with peers and siblings. Sometimes the success of older sisters or brothers put a lot of pressure on younger siblings to perform at a high level.
- Increased challenges may trigger MA.

 Some students reported loss of confidence when encountering more challenging work than before. This may have also happened when children were put into a higher achievement group than previously where expectations towards them were higher.

 "... in year 7 I was in the middle group, but I was top of the class... when she moved me up... my confidence just went straight down... because I realized how clever everyone else was in the top set, and how much more they learnt than me." Interview excerpt from a 12-13 year-old female student.
 - Older students (12-13 years of age) with high MA often referred to the increased hardness of math relative to their primary school experience as well as to increased homework load. They also often thought that stakes were higher in secondary than in primary school and this may have resulted in increased MA levels.
- Positive interpretation of experiences and resilience protect against MA.
 - In contrast to highly math anxious students, students with low MA often interpreted their negative experiences from a positive angle, often pointing to the importance of resilience and high self-efficacy in overcoming challenges faced by all children.
 - "sometimes my mind gets a bit confused ... I felt really frustrated ... but after two days ... everything went into my head and I knew everything." Interview excerpt from a 9-10 year-old female student.

In the classroom

- Assigning children to study groups must be done with care.
 The narrative of grouping should be positive, motivating lower achieving students to catch up. When moving students to higher achievement groups it should be clarified with them individually that expectations towards them are realistic.
- From time to time it may be advisable to create small work-groups
 of students with mixed abilities, when possible.
 Students with low abilities could benefit from working with higher
 achievers because peers with high achievement could explain their
 strategies to them. High achievers could benefit from working with
 lower achievers by improving their pro-social attitudes. Explaining
 strategies also improves meta-cognitive skill.
- Schools should aim to teach students methods of coping with increased stress before exams and when responding questions in front of a class.
 - Teachers should make sure that children do not intimidate their peers when making errors in front of the classroom. For example, it should be understood that errors inevitably happen from time to time and that they should be corrected in a constructive way. In general, no forms of bullying should be tolerated.
- Children should understand that responding to questions quickly is
 more of a requirement of good test results than good mathematics.
 Some students may be enchanted by discovering some
 mathematical regularities whereas they may dislike or may not
 understand the importance of working under time pressure.
 Teachers should help children to understand the distinction
 between fast solution speed (typically needed for good test results)
 and discovering mathematics (typically linked to creativity).
- The value of sustained and focused work should be emphasized.
 As the last interview excerpt above shows, perseverance is very important for progress.

Teachers and the school environment are crucial in shaping Math Anxiety

Teacher self-evaluation and teacher training are key in moderating MA.

Research findings

- Specific academic anxieties form by secondary school.
 As discussed in Section 3 MA becomes a more specific academic anxiety from primary to secondary school. Teachers are the most important persons and role models in this environment.
- Many primary school teachers may have high MA and this may transfer to their students.
 At least in the USA (mostly female) primary school trainee teachers reported the highest MA levels amongst college majors. Teachers' conscious or unconscious gender ability beliefs and MA affect students' gender ability beliefs and MA. When female teachers have
- Many more secondary than primary school students mentioned negative interpersonal relations with teachers as the cause of their MA.

high MA the performance of female students tends to decrease.

 In primary school a very frequently mentioned trigger of MA was when students were confused by different methods taught by different teachers or parents.

In the classroom

- Teachers should evaluate whether they themselves have high MA.
 Teachers should be aware that if they experience high levels of MA and strong gender ability beliefs towards mathematics that may negatively influence the mathematics performance of their students.
- Schools should consider it high priority to implement training programmes to close subject knowledge and teaching methodology training gaps.
 - Uncertainty in subject knowledge can induce MA in teachers. Training programmes could explicitly address the question of teacher MA (e.g., by honest group discussion). Some schools may struggle to hire well qualified math teachers. Such schools could organize sessions where more experienced or higher qualified colleagues can share their experience and subject knowledge. These events themselves should not become anxiety inducing triggers for teachers with lesser math subject knowledge. The objective should

- be to thoroughly and honestly discuss the to-be-taught material before classroom sessions happen.
- Teachers should treat challenging (potentially MA inducing) student questions as opportunity to grow and learn. Children will notice uncertainty and they will likely get confused by it. In the age of the Internet, it is very easy to find responses to any question online often from expert mathematicians (e.g., wikipedia. org or quora.com are excellent resources). This also offers unprecedented growth opportunities for math teachers.
- Teachers should assess their own communication skills and identify any gaps they may have.
 Teaching can be a hard and stressful job requiring constant and complex communication. Teachers should regularly evaluate their success in maintaining balanced communication and change their strategies if necessary.
- Teachers should aim to communicate fairly with children with the
 intention of solving problems.
 As do all people, teachers have their personal preferences. If
 teachers realize that they dislike a student, they should aim to
 understand their reasons for their own emotions and assure that
 their communication strategy will not disadvantage any student
 because of personal preferences.
- Schools should decide on best practice teaching methods and solution strategies to be taught to children.
 Some teachers may teach using better/more accessible methods than others depending on their subject knowledge and what they themselves have learnt during their training. The rationale for the preference for particular strategies should be discussed with children and clearly stated.

Suggested readings: Park, Ramirez, and Beilock, 2014; Carey, Devine, Hill, McLellan, and Szűcs, 2019.

Conclusions

High MA has short, medium and long-term impacts. On the short term student performance may suffer because task-irrelevant thoughts induced by MA interfere with performance. Such performance decrements may particularly affect more difficult problems that require more mental resources. This is in line with the fact that many otherwise reasonably well performing students show high MA and they rarely show very high levels of performance. On the medium term highly math anxious students will avoid elective math education opportunities in schools. This avoidance will obviously hold back their achievement level relative to their peers who do choose elective math classes. The lesser achievement levels can then further justify students' high MA. On the longterm math anxious students will avoid math-related, STEM career opportunities. Beyond childhood, severely math anxious adults may experience lower quality of life (e.g., just by the stress caused by quickly calculating sums in shops) and may avoid math-related situations with potentially negative outcomes (e.g., properly reflecting on their mortgage or credit card payments).

As shown, high MA can appear in diverse groups of students: First, low achievers with cognitive mathematics learning difficulties have about double the chance of developing high MA than other children. These children are in particular danger of developing a vicious circle between negative feedback from very low performance and high MA. Second, the overwhelming majority (close to 80%) of children with high MA are high achievers. While these children perform well on tests, their high MA may hold them back from optional/further math education and from choosing math-related careers even if they were interested in such careers and are perfectly able to study towards such careers. Finally, females are in particular danger of developing high MA. The most important reason for this may be the presence of engraved stereotypes about math not being a female domain. In addition, girls seem to be predisposed to higher (reported) anxiety levels in general. These higher anxiety levels may also predispose them to develop higher MA than boys. These higher anxiety levels are likely to be socially constructed in some manner but the exact mechanisms remain to be seen.

The above three groups (low achieving students with cognitive mathematics learning difficulties, high achieving students with high MA and girls) likely need different main interventions to prevent and alleviate their MA. Low achievers may benefit from very gradual performance build up to boost self-confidence and selfefficacy toward mathematics. Higher achievers with high MA may primarily benefit from increased meta-cognitve skills and will be able to separate their performance levels from their worries about mathematics. Some high achievers may need couching on anxiety management regarding public performance in class and some kind of confidence boost if they are placed into higher ability groups (maybe simply a serious reassurance on behalf of the teacher that the student has all skills to cope). Females may primarily benefit from breaking gender stereotypes about mathematics learning and career opportunities. Discussion of worries about math and their potential resolutions is useful to all groups experiencing MA. Interventions must be combined in some cases (e.g., in case of a high achieving girl with high MA).

Besides intervening at the individual student level it is essential that teachers clarify their own MA level, their gender ability beliefs about math learning, their student communication abilities and preferences as well as their potential subject knowledge gaps that can lead to math anxiety. Schools should organize teacher trainings where subject matter knowledge, best practice in teaching methods and communication and teacher and student MA are considered in a joint framework. Schools should decide on desired teaching and solution methods implemented in classrooms so that these are not confusing students potentially leading to MA. Teachers may find it important to clarify whether families show strong gender stereotyping of learning domains and whether they attribute low or high value to mathematics.

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